572

VOLUME 80 SEPARATE No. 572

PROCEEDINGS

AMERICAN SOCIETY OF CIVIL ENGINEERS

DECEMBER, 1954

DISCUSSION OF PROCEEDINGS - SEPARATES

262, 445

SOIL MECHANICS AND FOUNDATIONS DIVISION

Copyright 1954 by the AMERICAN SOCIETY OF CIVIL ENGINEERS
Printed in the United States of America

Headquarters of the Society
33 W. 39th St.
New York 18, N. Y.

PRICE $0.50 PER COPY

Current discussion of papers sponsored by the Soil Mechanics and Foundations Division is presented as follows:

Number		Page
262	Anchored Bulkheads, by Karl Terzaghi. (September, 1953. Prior discussion: None. Discussion closed)	
	Wu, T. H.	1
	Bigler, W. D.	4
	Hansen, J. Brinch	5
	Tschebotarioff, Gregory P.	7
	Baumann, Paul	16
	Briske, Rudolf	18
	Lea, Norman D.	19
	Packshaw, S.	21
	Verdeyen, J. and Roisin, V.	27
	Terzaghi, Karl (Closure)	31
445	Suction Force in Soils Upon Freezing, by Alfreds R. Jumikis. (June, 1954. Prior discussion: None. Discussion closed)	
	Krynine, D. P.	45

Reprints from this publication may be made on condition that the full title of paper, name of author, page reference (or paper number), and date of publication by the Society are given.

The Society is not responsible for any statement made or opinion expressed in its publications.

This paper was published at 1745 S. State Street, Ann Arbor, Mich., by the American Society of Civil Engineers. Editorial and General Offices are at 33 West Thirty-ninth Street, New York 18, N.Y.

Discussion of
"ANCHORED BULKHEADS"

by Karl Terzaghi
(Proceedings-Separate No. 262)

DISCUSSION

T. H. Wu,[30] J. M. ASCE.—In his comprehensive treatment of the subject of bulkheads, Mr. Terzaghi has drawn extensively from the results of field measurements and large-scale model tests. The writer takes this opportunity to correlate the author's data with the results of some measurements made on the bracings of an open cut for a depressed trainway in El Paso, Tex. This bracing system was essentially an anchored bulkhead.

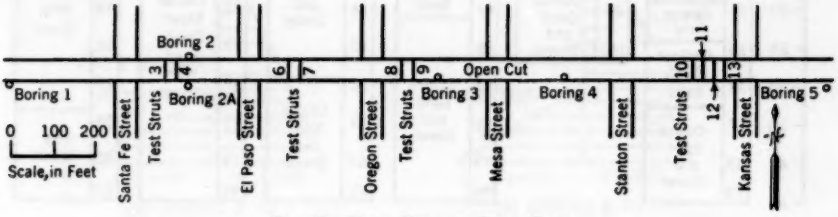


FIG. 17.—PLAN VIEW OF OPEN CUT

The open cut was 25 ft deep, and the bracing system consisted of a sheet-pile wall on each side of the cut supported at the top by continuous horizontal wales and cross struts. Fig. 17 shows the general layout of the open cut, and Fig. 18 shows the details of the bracing system.

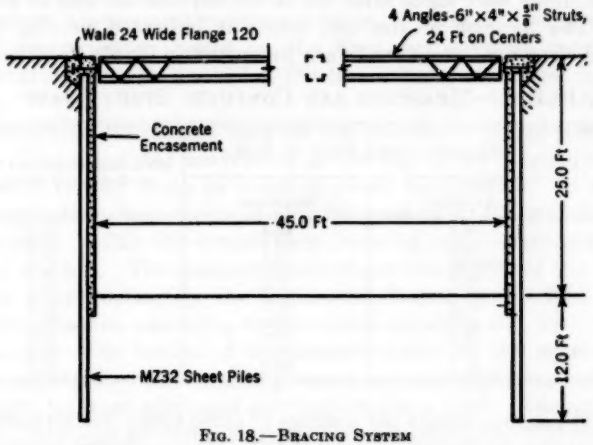


FIG. 18.—BRACING SYSTEM

The subsoil at the site was primarily sandy clay, coarse and medium sand, and gravel. The results of six borings are presented in Fig. 19. The values given in Fig. 19 in the columns under the letter "N" represent the number of blows per foot of penetration obtained by the standard penetration test.[24] In general the material west of Oregon St. was of medium density to a depth of

[30] Asst. Prof. of Civ. Eng., Michigan State College, East Lansing, Mich.

25 ft, and the standard penetration test resulted in 30 blows per ft. East of Oregon St. the material passed into the loose state, and the average number of blows was approximately 10. Below 25 ft the sand was dense; however, the average number of blows varied considerably.

The first step in the construction procedure was to drive the sheet piles and to excavate to a depth of 12 ft below ground level. The struts and wales

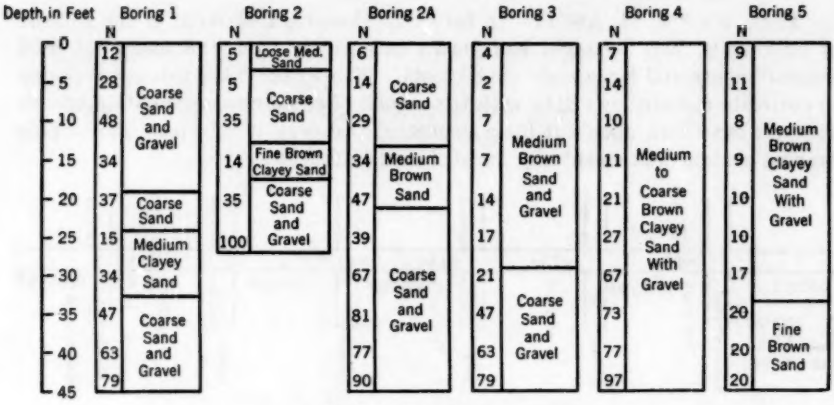


FIG. 19.—BORING LOGS

were installed, and the cut was excavated to its final depth. Strain measurements were made on both legs of the four angles comprising the strut. An 8-in. portable strain gage was used. No-load readings were made when the struts were on the ground and also when they were in place but not bolted; subsequent readings were made after the struts were bolted and as excavation progressed. The location of the test struts is indicated in Fig. 17. The measured and computed strut loads are presented in Table 3. The average

TABLE 3.—MEASURED AND COMPUTED STRUT LOADS

Strut No.	MEASURED STRUT LOAD, IN KIPS		Strut load computed from fixed-earth-support theory, in kips
	Average	Maximum	
3	48.4	48.4	67.5
4	48.4	60.5	67.5
6	50.2	50.2	67.5
7	50.2	55.4	67.5
8	57.0	65.7	94.5
9	65.7	81.3	94.5
10	43.2	43.2	94.5
11	70.7	91.6	94.5
12	76.1	86.5	94.5
13	81.1	88.2	94.5

measured strut load represents the average of three strain measurements made after the excavation of the cut was completed. The maximum measured strut load shown in Table 3 is the largest load recorded in those measurements.

The earth pressures, required depth of sheet-pile penetration, and strut loads were computed in the following manner: Values of K_A and K_P equal to

0.25 and 7.0, respectively, were assigned to the sand of medium density (east of Oregon St.). The subsoil west of Oregon St. was considered to be loose silty sand with values of K_A and K_P equal to 0.35 and 3.0, respectively. These coefficients of active and passive earth pressures were obtained from Table 2. The required depth of sheet-pile penetration, as determined from Eq. 17, was approximately 7.5 ft for the loose silty sand. The penetration provided in this open cut was 12 ft. The loads on the struts, which correspond to the anchor pull described by the author, were computed for the fixed-earth-support condition as defined in Fig. 1(*b*). The point of inflection of the sheet-pile wall was assumed to be 1.0 ft below the bottom of the cut. The computed strut loads are listed in Table 3, and the basis of the computations is shown in Fig. 20.

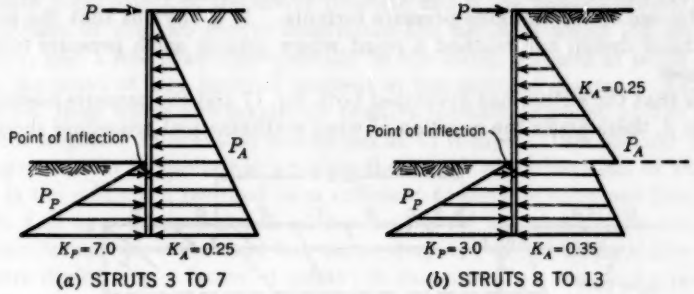


FIG. 20.—BASIS OF STRUT-LOAD COMPUTATION

Comparison of the measured and computed strut loads in Table 3 indicates that their agreement is satisfactory. The coefficients of active earth pressure, as deduced from the measurements, agree closely with those recommended by Mr. Terzaghi for loose silty sand and medium clean sand. It also appears that, when the sheet-pile penetration is increased to 1.5 times that required by Eq. 17, strut-load computations based on the fixed-earth-support condition would lead to better results than would Eq. 18 which is based on the free-earth-support condition.

Unfortunately, no reliable data on the movement of the sheet piles were obtained. The computed movement at the top of the sheet-pile wall for an excavation 12 ft deep (with no struts in place) was 0.008 ft. It was assumed in the computations that the sheet piles behaved as cantilevers resisting active earth pressure. After the struts were installed, no further movement was possible at the top. The computed movement was 0.0003 of the depth of the cut. This is approximately the movement that is required to develop the active earth pressure, according to the results shown in Fig. 3(*b*).

Conclusion.—The results of the measurements on the open cut indicate that the coefficients of active earth pressure are close to those recommended by Mr. Terzaghi for loose silty sand and medium clean sand. The order of magnitude of the sheet-pile movement agrees with the amount of movement required to develop active earth pressure as determined by the author's tests. The depth of sheet-pile penetration in this cut was 1.5 times the required amount, and the measured strut loads show that the behavior of the sheet pile was close

to the condition of fixed earth support. It is hoped that this information and correlation will be a contribution to the vast quantity of field data that are necessary to formulate design principles.

Acknowledgments.—De Leuw, Cather and Company of Chicago, Ill., designed the trainway and made the strut-load measurements with the assistance of C. P. Siess, M. ASCE, of the University of Illinois, at Urbana.

W. D. BIGLER.[31]—Mr. Terzaghi's studies have answered some of the questions concerning lateral earth pressures acting upon tied bulkheads. There is ample evidence from strain-gage and pressure-gage measurements made on harbor structures in San Pedro Bay (California) that the final lateral force caused by backfill in confined areas is substantially greater than that determined by use of the Rankine pressure formula. It is obvious that the science of bulkhead design has reached a point where greater earth pressure must be recognized.

Now that the author has developed both Eq. 17 and the pressure coefficients of Table 2, there arises the question of what mathematical procedure should be

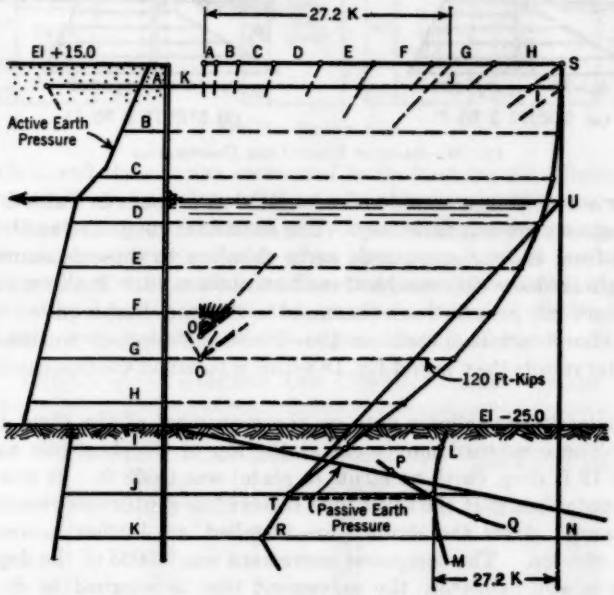


FIG. 21.—A METHOD FOR DETERMINING THE MAXIMUM BENDING MOMENT AND THE REQUIRED DEPTH OF PENETRATION

used in computing the maximum bending moments and the required depth of penetration. Fig. 21 illustrates a suggested convenient procedure. Unit earth pressures have been selected from Table 2.

In Fig. 21 the vertical line of the steel sheet-pile bulkhead is divided into spaces of conventient width (in this case a width of 5 ft). A force vector, A,

[31] Structural Engr., Long Beach, Calif.

B, . . . K, is computed for each space and placed at its midheight. Below the mud line, the vector is the algebraic sum of the active and passive pressures. The total available passive pressure is used to determine the bending moments.

The forces are drawn in the usual manner for graphically computing moments; the origin, 0, is placed directly below the left end of vector A; and the string polygon is drawn as shown. The abscissa at each point, such as line R–N, measures the cantilever moment caused by all earth forces above the point. This moment divided by the distance to the tie rod is equal to the tension required in the rod to maintain equilibrium. The tension is plotted with the vertical line S–N as a base to form the curve L–M.

Another curve, P–Q, is similarly drawn representing the shear above the reference point—that is, the active pressure minus the passive pressure. The intersection of the two curves determines the depth at which equilibrium is reached, and a line drawn horizontally to the string polygon at point T indicates the point of zero bending moment in the moment diagram. Line T–U is drawn, from which the maximum positive moment is scaled as 120 ft-kips, and the negative moment at the tie-rod as 45 ft-kips. This method of computing moments is applicable whether the length of embedment of the bulkhead is the minimum required or is sufficient to provide apparent fixity.

To find the required depth of penetration of the sheet piles, the earth-pressure forces below the mud line are recomputed on the basis of the passive pressure divided by a factor of safety; in this case G_s is assumed equal to 2. Likewise, the corresponding curves L–M and P–Q are reconstructed in a lower position. This final step, which is not shown in Fig. 21, indicates that the required penetration is 12.5 ft plus 20%, or 15 ft. Therefore, the value of H to be used in Eq. 12 is 52.5 ft.

Application of the moment-reduction curve of Fig. 16 indicates that a reduction of 27% is allowed. The final moment is therefore 0.73×120.0 ft-kips, or 87.6 ft-kips. If a steel-stress value of 22,000 lb per sq in. is used, the section modulus required is 47.8 in.^{3}, and a Z38 section (S = 46.8 in.^{3}) is required. The tie-rod tension is 27.2 kips per ft of bulkhead. Apparently the author's statement (under the heading, "Anchor Pull: Anchor-Pull Reduction") regarding reduction of anchor pull implies that reduction is not justified under usual conditions.

J. Brinch Hansen[32].—The errors involved in bulkhead design and the advances that have been made in this type of design have been clearly outlined by Mr. Terzaghi. The design procedure recommended by the author is a definite improvement on the methods which utilize only the classical earth-pressure distributions.

The decisive proof of the suitability of a design method, however, is its agreement with practical experience; in this connection the "Danish Rules" should be cited. This empirical method for the design of anchored bulkheads, which has been described[33,34] by Mr. Tschebotarioff, was developed by Danish

[32] Chf. Engr., Christiani and Nielsen, Copenhagen, Denmark.

[33] "Soil Mechanics, Foundations and Earth Structures," by G. P. Tschebotarioff, McGraw-Hill Book Co., Inc., New York, N. Y., 1951, p. 299.

[34] *Ibid.*, pp. 502–504.

engineers in 1926. Although numerous dredge and fill bulkheads designed from these rules have been constructed in water depths of up to 40 ft, no failures have been recorded except for a few cases of excessive anchor yield.

Although certain details in the Danish Rules can be criticized, it is a fact that, by this method, a great number of sufficiently safe and economical bulkheads have been constructed. On the basis of this extensive practical evidence, it is maintained that any design method which consistently leads to greater dimensions than those required by the Danish Rules must be considered uneconomical.

If a comparison is made between the Danish Rules and Mr. Terzaghi's proposed method of design, the following will be found: In the Danish Rules the factor of safety for the passive earth pressure is nominally 2, but is actually approximately equal to 1.4 whereas Mr. Terzaghi recommends safety factors of from 2 to 3 and (in addition) a 20% increase of the computed driving depth. The moment reduction allowed in the Danish Rules is dependent on the flexibility of the wall and is usually about the same as that found by Mr. Rowe,[19] whereas Mr. Terzaghi proposes to account for only half of this reduction. The smaller driving depth specified by the Danish Rules means a further reduction of the moments.

In a certain typical case, it was found that Mr. Terzaghi's procedure (with a safety factor equal to 2 and an allowable stress equal to 2/3 of the yield stress) required a 75% greater driving depth (corresponding to 15% longer sheet piles) and a 60% greater section modulus of the wall than the Danish Rules.

A common feature of both Mr. Terzaghi's design method and the Danish Rules is that certain allowable stresses are specified which are independent of the flexibility of the wall. On the basis of Mr. Rowe's investigations,[19] however, it is evident that a comparatively rigid wall must possess a greater reserve of safety against failure than a more flexible wall which has almost exhausted its possibilities of redistributing the earth pressures. Therefore, it might be logical to vary the allowable stress with the flexibility of the wall.

The problem of the design of flexible anchored bulkheads is evidently one in which proportionality between total load and maximum stress does not exist, and in such a case the usual concept of allowable stresses is not suitable. In other fields of engineering such problems are investigated by considering the state of failure; the structure is then designed so as to possess a certain safety against failure. Similar design methods are also used for structures (such as slabs and shells) which might have been designed by the application of the theory of elasticity.

In the case of bulkheads and similar structures, a computation based on a state of failure (for example, one in which a yield hinge is supposed to develop in the wall) has until recently not been possible because methods have not been available for determining the earth pressures corresponding to arbitrary movements of the wall. However, following the development and application of such methods of investigation to anchored bulkheads and other earth-retaining structures,[35] nothing should now prevent the engineer from designing such structures on the basis of a state of failure.

[35] "Earth Pressure Calculation," by J. Brinch Hansen, The Danish Technical Press, Engineering House, Copenhagen, Denmark, 1953.

The only shortcoming of such a design method is that it does not provide information about the actual deformations of the structure under normal working conditions. This is less important for earth-retaining structures than for almost any other structure, however, because the former structures can (without detrimental effects) suffer deformations of a magnitude which would be unallowable for most other structures. Consequently, bulkheads would seem to be ideal objects for computational methods based on the state of failure.

GREGORY P. TSCHEBOTARIOFF,[36] M. ASCE.—The need for a revision of the wasteful conventional methods of anchored-bulkhead design has long been apparent. Mr. Terzaghi is to be complimented for emphasizing this need and for presenting an over-all review of developments in this field. Unfortunately, however, the paper contains several misstatements and omissions.

The writer's procedure for the design of anchored bulkheads driven into clean sand is discussed in Part II (under the heading, "Computation of Maximum Bending Moments: Free Earth Support"). The author states:

> "Mr. Tschebotarioff has suggested computation of the maximum bending moment in the sheet piles on the assumption of fixed earth support irrespective of flexibility and of the relative density of the sand into which the sheet piles are driven.[18] Mr. Rowe showed that the errors involved in such a procedure can be very important and they are on the unsafe side[19]."

This statement is not correct. It should be noted that Mr. Terzaghi refers only to Mr. Rowe's original paper[37] according to which the writer's method results in bending moments approximately 25% less than those obtained by the Danish Rules. It was shown subsequently that Mr. Rowe was incorrect and that the reverse is true—that is, the Danish Rules result in bending moments 25% less than those from the writer's method.[38] Mr. Rowe's original figure was corrected accordingly, and Mr. Rowe accepted the correction.[39] This point is of importance because a large number of bulkheads have been built in accordance with the Danish Rules and have performed satisfactorily. Subsequent remarks will demonstrate a fallacy in the reasoning on which Mr. Terzaghi based this comment relative to the writer's computation procedure.

The physical significance of the flexibility coefficient ρ as presented in the diagrams offered by Mr. Terzaghi and Mr. Rowe cannot be visualized easily. Therefore, Fig. 22(*a*) shows the relationship between the values of log ρ as given by the author (and by Mr. Rowe) and the values of the coefficient ρ expressed in more consistent units—square inches per pound per foot—and to the total length H for five typical steel sheet-pile sections. The coefficient ρ has dimensions that are the reciprocal of stress. The units "per ft" merely indicate that the moment of inertia I must be computed for a wall width of 1 ft. In Fig. 22(*b*) there are shown the structural curves for the same five sheet-pile

---

[36] Prof. of Civ. Eng., Princeton Univ., Princeton, N. J.

[37] "Anchored Sheet-Pile Walls," by P. W. Rowe, *Proceedings*, Inst. of C. E., London, England, Vol. 1, Pt. 1, 1952, p. 35, Fig. 8.

[38] Correspondence of G. P. Tschebotarioff of "Anchored Sheet-Pile Walls," by P. W. Rowe, *ibid.*, pp. 616–619.

[39] Correspondence of P. W. Rowe of "Anchored Sheet-Pile Walls," *ibid.*, p. 633.

sections, Mr. Rowe's bending-moment-reduction curves, and the line representing the writer's design proposal.

This design-proposal line was obtained (to simplify comparisons) for the case in which α is equal to 0.7 and β equals 0.2, as used by Mr. Rowe.[37] It was further assumed that (1) the water level coincides with the anchor level; (2) the buoyant unit weight, γ', of the sand below the water level equals 0.6

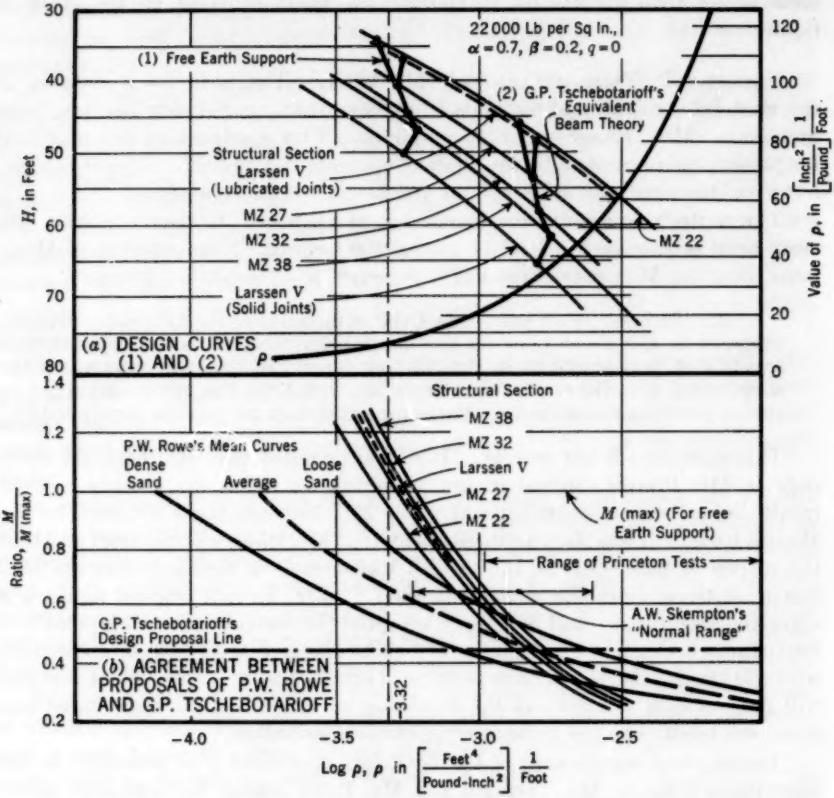


FIG. 22.—RELATIONSHIP BETWEEN H OR $M/M(\max)$ AND LOG ρ

times the weight, γ, of the sand above the water level; and (3) the surcharge is equal to zero. If H is taken in feet, the bending moment for free earth support, expressed in foot-pounds per foot, is

$$M(\max) = 0.0228\,\gamma\,K_A H^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

Similarly, the bending moment, M_T, used in the writer's design method which is illustrated in Fig. 23(*b*) is

$$M_T = 0.0102\,\gamma\,K_A H^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (20)$$

The ratio of M_T from Eq. 20 and of $M(\max)$ from Eq. 19 equals 0.45, which value is plotted as a straight horizontal line in Fig. 22(*b*).

It should be noted that in Fig. 22(*b*) the average of the structural lines for the steel sheet piling, the design line proposed by the writer, and Mr. Rowe's mean curve for medium sand all intersect at approximately the same point. Thus, according to Mr. Rowe's data, the design proposal shown in Fig. 23(*b*) gives the strength required by a maximum tolerable flexibility of steel sheet piling. The structural curves for stiffer steel bulkheads (to the left of the intersection) will all be higher than Mr. Rowe's mean curve for medium sand and will therefore have a strength in excess of the increased strength required by their increased stiffness. Mr. Rowe selected a model bulkhead with the

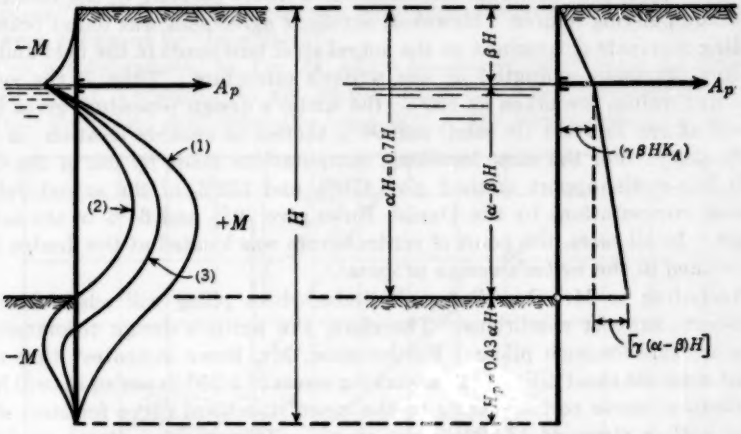


FIG. 23.—BENDING-MOMENT CURVES AND DESIGN METHOD

values of α equal to 0.7 and β equal to 0.2 to demonstrate the fact that a stiff bulkhead with log ρ equal to -3.32 was subjected (during his model tests with loose sand) to a bending moment approximately twice the value of the moment computed according to the writer's method (Fig. 23(*b*)). This fact is not questioned; however, it is a fallacy to believe that such a bulkhead would be unsafe.

Example.—A wall made of MZ 38 steel sheet piling[40] that is 45 ft long (I equal to 281 in.^{4} per ft) has a flexibility coefficient ρ equal to 10.2 in.^{2} per lb per ft, and log ρ equal to -3.32. From Eq. 19—for K_A equal to 0.333 and γ equal to 105 lb per cu ft—M(max) is equal to 873,000 in.-lb per ft. According to Fig. 22(*b*) and Mr. Rowe's mean curve for loose sand, one should use 85% of M(max) or 742,000 in.-lb per ft. At a working stress of 22,000 lb per sq in. (that is, at $\frac{2}{3}$ of the yield stress, the minimum suggested by Mr. Terzaghi) an MZ 38 section (Z equal to 46.8 in.^{3} per ft) resists 1,030,000 in.-lb per ft, or 118% of M(max) (Fig. 22(*b*)). This is appreciably more than is needed

[40] "Carnegie-Illinois Steel Sheet Piling. Insert C, Z Piles," Carnegie-Illinois Steel Corp., Pittsburgh Pa., February, 1936.

according to Mr. Rowe's proposals. However, the use of the writer's design proposal for H equal to 45 ft, as shown in Fig. 22(a), would lead to the selection of the much more flexible and lighter MZ 22 section.

Thus, the writer's design proposal cannot be unsafe, and Mr. Terzaghi's use of Mr. Rowe's original paper[19] as a basis for his criticism is unjustified where steel sheet piles are concerned.

The design proposals shown in Fig. 23(b) have been substantiated by full-scale field measurements on steel sheet piling in the harbor of Bremen, Germany, by Dietrich Wiegmann.[41] The double anchorage—at two levels—of that steel sheet-pile wall does not allow for a direct plotting of the results on the accompanying figures. However, excellent agreement was found between bending moments determined on the actual steel bulkheads in the field and the bending moments computed by the writer's procedure. Thus, if the actual measured values are taken as 100%, the writer's design procedure gives 98% thereof at one location (in sand) and 96% thereof at another location (in stiff sandy clay). For the same locations, computations made by use of the Coulomb free-earth-support method give 170% and 133% of the actual values, whereas computations by the Danish Rules give 70% and 64% of the actual values. In all cases, the point of contraflexure was located at the dredge line, as assumed in the writer's design proposal.

According to Mr. Rowe's tests,[19] timber sheet piling will adapt itself to fixed-earth-support conditions. Therefore, the writer's design recommendations are safe for such piling. Furthermore, Mr. Rowe indicates[42] that reinforced concrete sheet piling with a working stress of 1,250 lb per sq in. will have a structural curve corresponding to the mean structural curve for steel sheet piling with a stress of 17,900 lb per sq in. (This working stress would be assigned in the United States to concrete having a 28-day strength of f'_c equal to 2,780 lb per sq in.) Hence the writer's findings, relative to steel sheet piles, are applicable to concrete of this quality which should normally be expected in present-day (1954) waterfront structures.

Fig. 16 shows that, for any concrete, tolerable moment reductions should be smaller than for steel. This is incorrect as Fig. 16 is applicable only to reinforced concrete sheet piles made of concrete having a very low strength (such as concrete having a 28-day strength of f'_c equal to 1,670 lb per sq in.) in which case only, according to Mr. Rowe,[42] any appreciable deviations from the writer's design proposals are to be expected. However, concrete of such poor quality should never be used in any properly designed waterfront structure. Such poor concrete may have been used in the first applications of reinforced concrete; yet early Danish bulkheads—although designed for bending moments at least 25% less than those which follow from the writer's design procedure[43, 44]—have performed satisfactorily. Thus, the writer's design proposal

[41] "Der Erddruck auf Verankerte Stahlspundwände ermittelt auf Grund von Verformungsmessungen am Bauwerk," by Dietrich Wiegmann, dissertation approved by the Technische Hochschule (Technical University) of Hannover, Hannover, Germany, in 1953, for the award of the degree of Doktor-Ingenieur.

[42] "Anchored Sheet-Pile Walls," by P. W. Rowe, *Proceedings*, Inst. of C. E., London, England, Vol. 1, Pt. 1, 1952, p. 57, Fig. 21.

[43] "Soil Mechanics, Foundations and Earth Structures," by Gregory P. Tschebotarioff, McGraw-Hill Book Co., Inc., New York, N. Y., 1951, pp. 500–501.

[44] *Ibid.*, pp. 540–542.

shown in Fig. 23(*b*) is entirely safe as evidenced by the field performance of reinforced concrete sheeting.

The agreement (illustrated by Fig. 22(*b*)) among the structural curves for steel sheet piles, Mr. Rowe's mean curves, and the writer's design proposal is not accidental. The effects of the flexibility of sheet piles have been established and were taken into account by the writer's design proposal.[18] A contrary implication by Mr. Terzaghi is incorrect, as is the statement in Part I (under the heading, "Influence of Flexural Rigidity on Bending Moment") that "no observational data were available" concerning the effects of bulkhead flexibility "* * * until the results of Mr. Rowe's experimental investigations were published.[19]" Relevant observational data were obtained prior to Mr. Rowe's and were published by the writer.[18]

This is substantiated by Fig. 24, which reproduces data[45] from the report[18] quoted by Mr. Terzaghi. In Fig. 24 the test numbers in parentheses indicate the tests performed with originally loose sand. The same bulkhead thickness

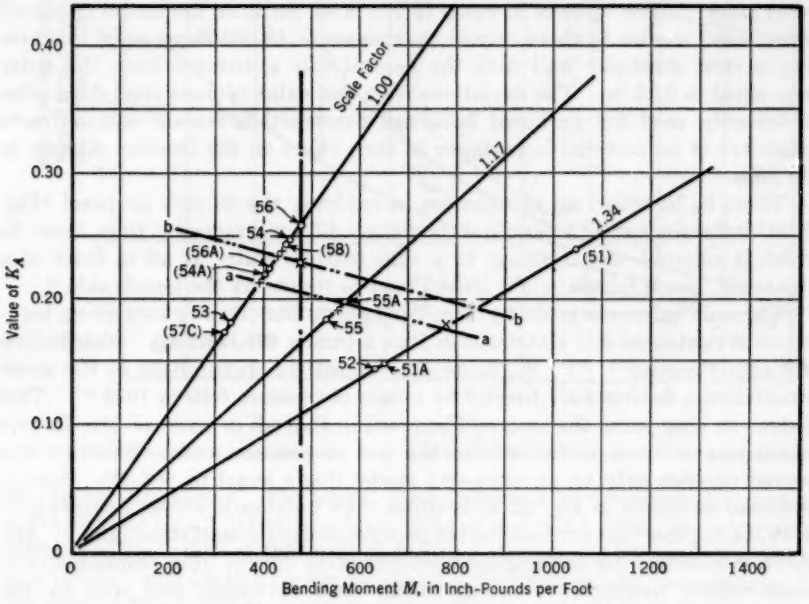


FIG. 24.– COMPARISON OF MEASURED BENDING MOMENTS AND DESIGN METHOD OF FIG. 22(*b*)

was used in all tests, but the vertical dimensions were varied according to three different scale factors. Thus (for α equal to 0.612), the scale factor equal to 1.00 corresponded to H equal to 73 in.; the scale factor equal to 1.17 corresponded to H equal to 83 in.; and the scale factor equal to 1.34 corresponded to H equal to 98 in. Most of the model bulkheads had an α-value of 0.612 $\left(\frac{H_p}{\alpha H}\right.$ equal to 0.63$\left.\right)$. Tests 54 and (54A) had α equal to 0.700 $\left(\frac{H_p}{\alpha H}\right.$

[19] "Final Report. Large Scale Earth Pressure Tests with Model Flexible Bulkheads," by G. P. Tschebotarioff, Princeton Univ., Princeton, N. J., 1949. Fig. 42.

equal to 0.43) and H equal to 65 in. Tests 56 and (56A) had α equal to 0.786 ($\frac{H_p}{\alpha H}$ equal to 0.27) and H equal to 57 in. The three inclined straight lines in Fig. 24 are the loci of bending moments for the bulkheads of the three flexibilities computed in accordance with the procedure shown in Fig. 23(*b*), as well as the K_A-values corresponding to the bending moments measured during the actual tests. The effect of increasing flexibility is shown by the fact that smaller values of K_A correspond to tests where the bulkhead had higher scale-factor values—hence, greater lengths and flexibilities. The line a–a corresponds to the average values of bulkhead tests with α equal to 0.612; the line b–b expresses the probable similar relationship for α equal to 0.700. The intersection between line b–b and a vertical line drawn through M equal to 478 in.-lb per ft results in a design value of K_A equal to 0.23 for model conditions. This value was then increased to compensate for a difference in the effects on the model and on the prototype of capillary saturation of the sand above water level.[18] The value of M equal to 478 in.-lb per ft on the model produced deflections[18] similar to those caused by stresses of 18,000 lb per sq in. in a prototype steel sheet-pile wall with the neutral axis a distance from the outer edge equal to 5.75 in. The deviations from this value of most steel sheet piles customarily used for anchored bulkhead construction remain within limits which are of no material importance in their effect on the bending stresses in the piles.

There is, however, no justification in applying the writer's proposal (Fig. 23(*b*)) indiscriminately to unusual conditions differing radically from those to which it referred—for instance, to a case with no water at all in front of a bulkhead. Such misuse might indeed lead to results on the unsafe side.[42]

The only reference made by Mr. Terzaghi to Mr. Rowe's measuring techniques is contained in a statement in Part I (under the heading, "Distribution of Earth Pressure"): "P. W. Rowe determined the distribution of the earth pressure on a flexible wall directly by means of pressure cells in 1952.[19]" This statement may leave the incorrect impression that all or most of Mr. Rowe's measurements were performed by the use of pressure cells. Actually this proved possible only on an ultra-stiff model ($\log \rho$ equal to -3.32). Such a bulkhead as shown in Fig. 22(*a*) is stiffer than bulkheads which, according to A. W. Skempton[46] have a "normal range of flexibility for steel sheet piles." Mr. Rowe established his bending-moment-reduction curves from bending-strain-measurement techniques identical to the ones developed and used by the writer,[18] and not by pressure-cell measurements. The point is of importance for the understanding of this entire problem.[47]

Mr. Terzaghi and Mr. Rowe both overemphasize the effect of the original density of the sand and underemphasize the effect of variations in the depth of embedment of the piling. This is not justified.

Fig. 25(*a*) shows the alleged complete agreement between the writer's tests[18] and Mr. Rowe's mean curves as reported by Mr. Rowe.[19] Fig. 25(*b*) shows the actual relationship. Test 56 was omitted in Fig. 25(*a*), and the

[46] "General Report Session 7: Earth Pressures," by A. W. Skempton, *Proceedings*, Third International Conference on Soil Mechanics and Foundation Eng., Zürich, Switzerland, Vol. II, 1953, p. 354.

[47] Discussion on Session 7 (Earth Pressures) by Gregory P. Tschebotarioff, *ibid.*, Vol. III, 1953.

actual positions of tests 54 and (54A) are the reverse of the ones indicated by Mr. Rowe. This means that originally dense sands may lead to higher bending moments in the piling than originally loose sands. This seeming paradox is caused by the fact that the lateral resistance of a very loose submerged sand in front of the buried part of a bulkhead is not directly proportional to the initial density and to the static compression in a confined test of the sand, as

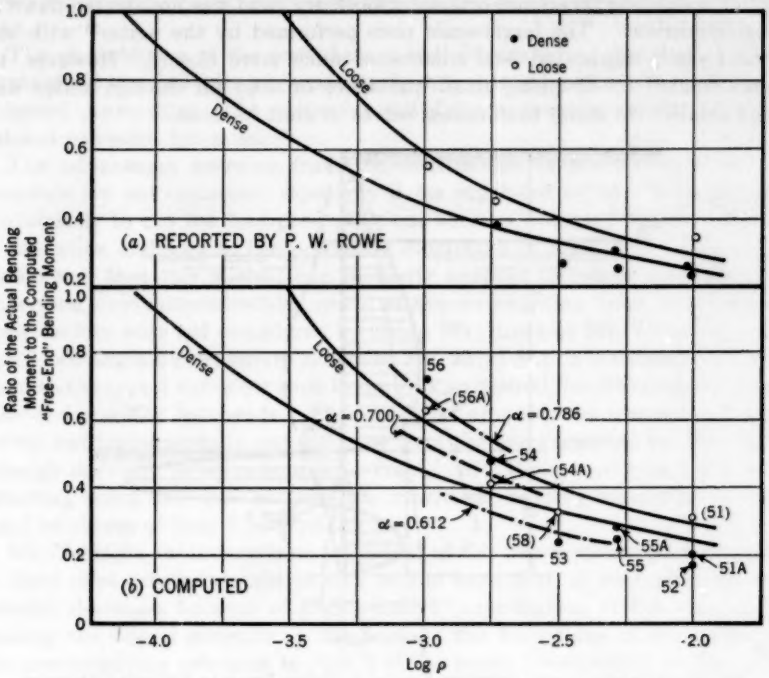


FIG. 25.—COMPARISON OF PRINCETON TEST RESULTS WITH P. W. ROWE'S MEAN CURVES

Mr. Rowe suggested.[19] The lateral yield induces a simultaneous and appreciable vertical consolidation of a large mass of such sand under its own weight and under the action of the vertical shearing stresses, which are transmitted to it through wall friction by the bulkhead. This fact can be demonstrated in any seepage flume with transparent walls. The lateral bulkhead displacements y_D at the dredge line (Fig. 26) were only 75% larger during tests (54A) and (56A) with originally loose sands than the displacements y_D during tests 54 and 56 with originally dense sands.[18] The maximum displacements did not exceed $\frac{1}{4}$ in. on the model—approximately 2.5 in. on a prototype—a value which cannot be considered excessive. According to Mr. Rowe's confined tests, loose dry sand should compress 4.3 times as much as dense sand[19]—that is, 2.5 times more than the 75% higher value of lateral yield actually observed for submerged sand.

The actual effect of variations in the depth of embedment is indicated in Fig. 25(*b*). It is evident that with an increasing α-value—that is, with a

decreasing depth of embedment—the moment reduction also decreases much more considerably than would follow from Mr. Rowe's mean curves shown in Fig. 13.

These discrepancies have the following explanation: Mr. Rowe performed his tests with dry sand only, and with a scale sufficiently small (H equal to 42 in.) to permit avoiding any shocks or vibrations during laboratory "dredging" or backfilling; therefore originally loose dry sand was not shaken down by these operations. The larger-scale tests performed by the writer[18] with submerged sands duplicated field conditions much more closely. However, the waves created by dropping small quantities of sand fill through water were much smaller (to scale) than actual waves in most harbors.

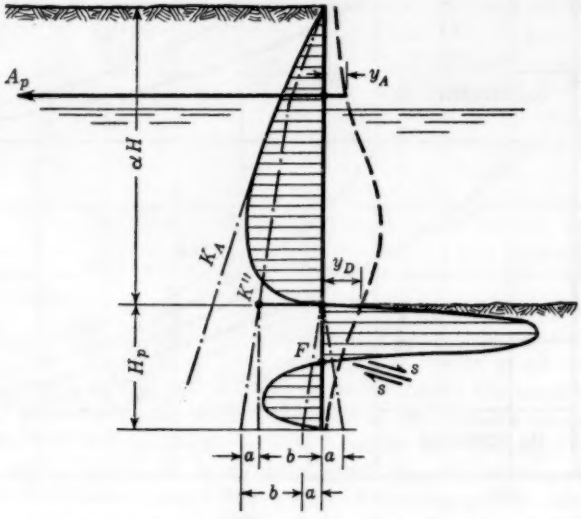


FIG. 26.—MAXIMUM EARTH PRESSURE VALUES, b, WHICH PROVIDED EFFECTIVE FIXATION OF TOE OF MODEL BULKHEAD

Except for the preceding reservations, the agreement between the results of Mr. Rowe's tests and the writer's is excellent. An examination of earlier summaries of the writer's findings[18, 48, 49] shows that the writer's tests established a number of points of fundamental importance for the analysis of bulkhead behavior; these points form the basis of Mr. Rowe's investigational procedures and findings. Some of these points are: (1) The breakdown of "arching" as a result of anchor yield; (2) the decrease of bending moments caused by the concentration of the passive pressures just below the dredge line, and by the decrease of active pressures just above the dredge line; and (3) the dependence of these pressure changes (at that boundary) on the bulkhead flexibility as it affects bulkhead displacements at the dredge line.[18, 48, 49] These limitations do not detract from the value of Mr. Rowe's achievements.

[48] "Soil Mechanics, Foundations and Earth Structures," by Gregory P. Tschebotarioff, McGraw-Hill Book Co., Inc., New York, N. Y., 1951, pp. 297–303, Figs. 10–41.

[49] *Ibid.*, pp. 505–516.

The writer has already welcomed[38, 47] Mr. Rowe's outstanding work both as a confirmation and as a development and extension of the writer's investigations. The writer recommends the use (with consistent units) of the coefficient ρ which Mr. Rowe introduced to define bulkhead flexibility. However, Mr. Terzaghi's suggestion that Mr. Rowe's bending-moment-reduction curves be used as a direct basis for field design ignores the fact that these curves reflect purely laboratory (stable) conditions and are not adapted to reflect field conditions.

The unwieldiness of the procedure is well illustrated by the fact that Mr. Terzaghi did not grasp its physical significance; hence he was led to the fallacy explained previously. The writer himself did not notice a similar fallacy at first and conceded too much.[38]

The advantages accruing from the routine use of Mr. Rowe's complex procedure are not apparent; especially if (as suggested by Mr. Terzaghi) one is arbitrarily to cut the moment reductions thereby obtained by one half.

Attention is drawn to the procedure illustrated in Fig. 23(*b*). The writer has shown[47] that this method can be easily adapted to reflect new field data concerning field imponderables, such as shocks resulting from wave action. These factors were not considered by either Mr. Rowe or Mr. Terzaghi. The type 2 bending-moment curves shown in Fig. 23(*a*) can be increased[47] to correspond to the type 3 curve (or type 1 curve), if so desired, for differing soil conditions or for stiffer bulkheads. This modified procedure is simpler and more flexible but fundamentally not different from the one suggested by Mr. Rowe, although the order of operations is reversed. Mr. Rowe used type 1 curves as a starting point and then reduced the maximum bending moments to correspond to curves of type 3 or type 2.

Mr. Terzaghi recommends no reduction of the free-earth-support moments for sheet piles driven into silt or clay on the basis that, in such soils, the end restraint decreases because of a progressive consolidation of the clay or silt resisting the lateral pressure on the back of the buried toe of the bulkhead. The accompanying reference to Part I of the paper (presumably to Fig. 1(*b*)) suggests that fixed-earth-support conditions occur only when the lower portion of the sheet pile has moved backward from its original position. This is a "classical" assumption which is tied to the belief that end restraint requires the mobilization of earth pressures of "passive" intensity. The writer's tests[18] have shown, however, that, in the case of sands, restraint of the toe of the bulkhead occurs even when it has moved out in the manner indicated in Fig. 1(*a*) and Fig. 26.

An analysis[50] has shown that the "classical" assumption that passive earth pressures have to be mobilized at the toe of a fixed-support bulkhead is incorrect. This was shown by the K''-values determined in accordance with Fig. 26. In all static tests with a depth of embedment corresponding to α equal to 0.612 and $\frac{H_p}{\alpha H}$ equal to 0.63, the K''-value varied between 0.07 and 0.23 with an average value for eight tests of K'' equal to 0.14. The effects of severe, delib-

[50] "Einfluss der Gewölbebildung auf die Erddruckverteilung," by Gregory P. Tschebotarioff, Dissertation approved by the Rhine-Westfalian Technische Hochschule (Technical University) at Aachen, Germany, in 1952, for the award of the degree of Doktor-Ingenieur.

erately produced vibration increased this value to $K'' = 0.23$. Thus, active pressures alone were sufficient to provide full end restraint for that particular depth of embedment. Recomputation[18] of the measured bending moments confirmed the validity of the pressure b from which these K''-values were computed. Even smaller K''-values will be obtained if they are referred to the $(a + b)$-pressures from a consideration of the shearing stresses S indicated in Fig. 26. These shearing stresses must decrease the active pressures a on the outer face of the buried portion of the bulkhead.

For tests 54 and (54A), with α equal to 0.7 and $\frac{H_p}{\alpha H}$ equal to 0.43, an average value (for the pressure b) for K'' of 0.45 was obtained. This value corresponds to earth pressures at rest. Only for the small depth of embedment (inadmissible in practice) of tests 56 and (56A) with α equal to 0.786 and $\frac{H_p}{\alpha H}$ equal to 0.27 was K'' equal to 0.85. This value is much smaller than the customarily assumed intensities of passive earth pressure.

Active and even neutral earth-pressure values can be relied on in clay and in silt soils. Thus, there is every reason to believe that appreciable end restraint will be permanently present in deep deposits of medium to stiff clay and silt, once a sufficient depth of bulkhead embedment is provided.

In discussing clay and silt soils, Mr. Terzaghi does not mention the use of a sand dike for the purpose of reducing fluid pressures of cohesive backfills. The writer's tests showed that such sand dikes are very effective when placed between a bulkhead and its fluid cohesive backfill.[18, 42, 49] The excavation of soft clay below the dredge line and its replacement by sand fill is also an effective measure[48, 49] since it provides a practical alternative for the use of the free-earth-support method which is frequently not feasible in such cases.

PAUL BAUMANN,[51] M. ASCE.—An excellent treatise on anchored bulkheads has been presented by Mr. Terzaghi. The clarity with which the complexities inherent in bulkhead design are expounded is commendable. This paper should serve to dispel the idea, current among many engineers, that a bulkhead is just another retaining wall. The fact is that a bulkhead has little in common with the conventional retaining wall. A retaining wall is a simple, statically determinate structure, depending primarily on gravity for its stability. The anchored bulkhead, however, is a statically indeterminate structure, depending for stability primarily on its anchorage (that is, its restraint) at the lower end, its anchorage at or near the upper end, and the strength of the sheet piles.

The majority of bulkhead failures have resulted from faulty anchorage. Hence, the predomiant cause of failure has been in the foundation. The author stresses the need for exploratory borings to ascertain the possibility of critical changes in subsoil conditions. Unfortunately this practice is sometimes considered an unnecessary expense by engineers. Such borings should also serve to ascertain the density, the shearing strength, and the stress-strain relationship of the soil. This stress-strain relationship is termed (perhaps somewhat misleadingly) the "modulus of elasticity," so far as the linear part of this relation is concerned.

[51] Asst. Chf. Engr., County Flood Control Dist., Los Angeles, Calif.

It has been shown[52] that, by treating a sheet-pile bulkhead as a statically indeterminate structure and by evaluating the specific, passive resistance (e_f) as a function of both the depth and the stress-strain relationship—everything else being equal—relatively rigid sheet-piles require a greater depth of penetration (anchorage) than relatively flexible ones. Mr. Terzaghi, in Part II (under the heading, "Safety Requirements: Allowable Fiber Stresses in Sheet Piles"), suggests that the friction in the interlocks of steel sheet piles should be based on the assumption of lubricated conditions—that is, on the basis of a relatively great flexibility number. As H and E are constant for the identical wall, it follows that ρ varies inversely with I. Hence, if the friction in the interlock approached a "frozen" condition, the flexibility number would be much smaller and the rigidity of the sheet-pile wall would be much greater than that assumed. The depth of penetration might therefore be rendered unsafe.

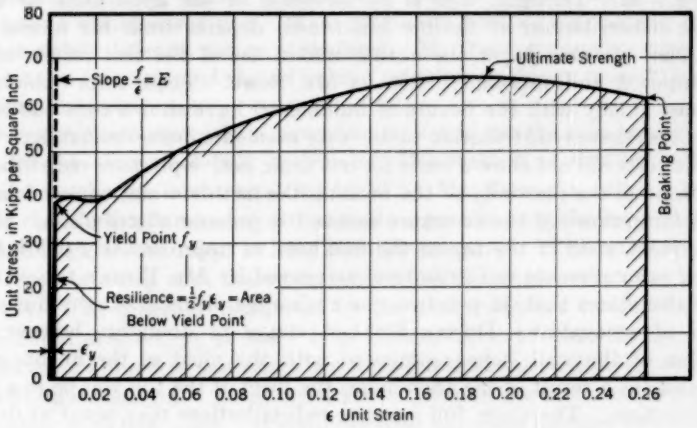


FIG. 27.—STRESS-STRAIN DIAGRAM FOR MEDIUM CARBON STEEL

In consequence, it is considered better practice to design a sheet-pile bulkhead on the assumption of maximum, rather than minimum, rigidity. However, this procedure may actually result in higher bending stresses than those analyzed. The reason for this assumption of maximum rigidity is the lack of known bending failures in steel sheet-pile bulkheads whereas failures caused by insufficient penetration have been almost as frequent with this type of bulkhead as with any other type.

When compared with other sheet-pile materials, steel has qualities which make it outstanding. The stress-strain relationship of medium carbon steel shown in Fig. 27 will serve to substantiate this statement. The yield point strength of this steel is approximately 40,000 lb per sq in., and the ultimate strength is approximately 68,000 lb per sq in. As Mr Terzaghi states, the allowable design stress should not exceed two thirds of the yield-point strength—that is, 26,700 lb per sq in. Hence, before the yield point is reached, the maximum bending moment would have to be 50% greater than analyzed, and the

[52] "Analysis of Sheet-Pile Bulkheads," by Paul Baumann, *Transactions*, ASCE, Vol. 100, 1935, pp. 792–797.

energy of deformation (the resilience) would then be 34 in.-lb per cu in. This value is quite small when compared with the 16,000 in.-lb per cu in. of energy required to reach the ultimate strength. Therefore, to cause structural failure, the energy required would be at least 700 times the energy associated with an allowable stress of 26,700 lb per sq in. The availability of such energy (cross-hatched area in Fig. 27) is, of course, unthinkable.

The foregoing remarks should not be construed to suggest the underdesigning of sheet-pile bulkheads. They are merely intended to show that anchorage failures are much more likely to occur than structural failures, so far as steel sheet-piles are concerned. In brief, it may be said that, in respect to structural qualities, there are steel sheet-piles and others.

RUDOLF BRISKE.[53]—In Part I (under the heading, "Distribution of Earth Pressure") Mr. Terzaghi, who is an advocate of the application of earth-pressure redistribution at flexible bulkheads, departs from his former considerations. In this paper the author makes use of the test results of Mr. Tschebotarioff and, especially, those of Mr. Rowe. These tests (which were concerned chiefly with the flexure of bulkheads) have shown that one cannot rely on a pressure redistribution to decrease moments indiscriminately. However, the tests did not show a basis for not using active-pressure redistribution.

Because of the plasticity of the backfill, the last deformation—and not the total deformation—of the structure causes the pressure distribution. Hence, a subsequent yield of the top of the bulkhead of approximately 0.001 H will cancel a prior pressure redistribution, as proved by Mr. Rowe's tests.[54] The author also states that, in practice, the anchorage yields gradually during the process of excavation. During the last stages of dredging, however, the deflection of the wall (when compared with the yield of the anchorage) is large, and (as a result of the anchorage) the yield of the top will be in a backward direction. Therefore, full pressure redistributions may occur at dredged bulkheads; also, at backfilled bulkheads, redistributions may be produced if the last critical deformation is an additional deflection of the bulkhead.[55] The total yield of the anchorage may be greater than the limiting value of 0.001 H if the last critical yield of the top of the bulkhead is smaller than this amount or is directed backward. By prestressing the anchorage, these conditions can be enforced.[55,56] Furthermore, a pressure redistribution will occur if the bulkhead yields plastically at a small factor of safety for the sheet piling.[55,56]

It is often stated that the pressure redistribution does not remain constant because the anchorage yields during the lifetime of a structure. However, a subsequent yield of the anchorage will cause an increase—or at least no decrease—of the anchor pull. In this manner, pressure redistribution is stabilized because anchor pull and bending moments are reciprocally dependent. That is, there will be a stable equilibrium between the increased anchor pull and the decreased bending moment.[55,56] Thus, a subsequent anchor yield

[53] Engr., Berlin, Germany.

[54] Correspondence of "Anchored Sheet-Pile Walls," by P. W. Rowe, *Proceedings*, Inst. of C. E., London, England, Vol. 1, Pt. 1, 1952, pp. 616–647.

[55] "Erddruckverlagerung bei Spundwandbauwerken," by Rudolf Briske, Wilhelm Ernst and Sohn, Berlin, Germany, 1953.

[56] Discussion on Session 7 by Rudolf Briske, *Proceedings*, Third International Conference on Soil Mechanics and Foundation Eng., Vol. III, Zurich, Switzerland, 1954.

will not cause a diminution of pressure redistribution if the anchor pull does not decrease.

All bulkheads at which earth-pressure redistribution was accounted for have performed satisfactorily when the anchorage was safely constructed. Therefore, a German committee for quay walls has recommended reduction of the moments by one third, if the bulkheads are computed on the basis of a classical pressure distribution (that is, without redistribution of active pressures resulting from a bulkhead deflection).[57] The problem of flexure has been thoroughly investigated, but it is dangerous to neglect the problems of active-pressure redistribution. The cases in which redistribution can be accounted for may be as many as those in which flexure occurs. Which type of structure should be erected is a matter of economy.

NORMAN D. LEA,[58] J. M. ASCE.—The subject of anchored bulkheads has long been a disorganized one. Many engineers hold widely varying opinions and use widely different design procedures. Mr. Terzaghi's paper consolidates the gains in this field and should help in establishing design procedures and in planning research.

Irregular Surcharges.—It is common practice to use the area behind a bulkhead for the storage of bulk materials. The method of dealing with these irregular surcharges is not explicitly cited by Mr. Terzaghi. One is tempted to add the effects of a number of line loads approximating the irregular part of the surcharge to the effect of the uniform surcharge. The methods recommended for working with these two types of surcharge, however, are basically different. The procedure suggested by the author for line (or point) loads is independent of the type of soil. It is known, however, that the type of soil does have an influence; this effect is considered in the procedure recommended for uniform surcharges. Thus, one is led to question the recommended method for dealing with line loads. It certainly does not seem justified to extend this method for use with irregular surcharges.

The effect of the yield ratio is important. The tests performed[10, 11] by Messrs. Gerber and Spangler permitted little, if any, yield. Therefore, it is not surprising that the resulting pressures were approximately twice those determined by use of the Boussinesq theory[12] and that the influence of the type of material was small. With a greater yield, pressures reduce (in some instances) to much less than the Boussinesq value. There is some evidence, from the performance of structures, to indicate that bulkheads located in granular materials yield sufficiently to effect a permanent reduction of the pressure.

Thus, the writer suggests that, for irregular surcharges placed on granular materials, the Coulomb method should continue to be used, whereas twice the Boussinesq pressure should be used in cohesive materials.

---

[57] "Technischer Jahresbereicht über die Arbeiten des Ausschusses zur Vereinfachung und Vereinheitlichung der Berechnung und Gestaltung von Ufereinfassungen im Rahmen der Hafenbautechnischen Gesellschaft und der Deutschen Gesellschaft fur Erd- und Grundbau," by E. Lackner, *Bautechnik*, Vol. 12, 1953, pp. 369–373.

[58] Assistant to the Vice-Pres., Foundation of Canada Eng. Corp., Ltd., Montreal, Que., Canada.

Lateral Earth Pressure in Cohesive Soils.—Mr. Terzaghi has been conservative in dealing with line loads; he has assumed practically no yield. In treating lateral earth pressures in cohesive soils, however, the reverse—unlimited yield—has been assumed; this may not be sufficiently conservative. The formula given in Table 2 for K_A in silt and clay assumes a full mobilization of the shear strength; this requires considerable yield. The writer has measured[59] such a yield of $\frac{1}{2}\%$, and it is conceivable that in some instances the yield necessary to mobilize the full shear strength may be at a rate of $\frac{1}{2}\%$ per year.

A further reason for being conservative in computing K_A for clays is that the formula in Table 2 will frequently result in a negative value. If, in such an instance, K_A is assumed to be zero, an unknown factor of safety is introduced. A further unknown factor of safety is introduced by neglecting the wall friction.

It is suggested that a factor of safety of 2 be introduced into the expressions given in Table 2: Thus,

$$K_A = 1 - \frac{q_u}{2(\bar{p} + \gamma z)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21a)$$

and

$$K_P = 1 + \frac{q_u}{2(\bar{p} + \gamma z)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21b)$$

Eqs. 21 should account conservatively for increased pressures caused by limited yield. Eq. 21*a* will also give a better approach to the problem in which K_A would have a negative value if the form shown in Table 2 were used.

Maximum Bending Moment.—In Part II (under the heading, "Computation of Maximum Bending Moments: Moment Reduction Resulting from Flexibility Effects") Mr. Terzaghi states "The first step consists of computing the maximum bending moments for free earth support * * *." It is presumed, although not stated by the author, that the passive resistance and pile length for this computation are determined for a factor of safety of unity. The writer suggests that for cohesive soils a factor of safety of 2 be introduced by using K_P from Eq. 21*b* in computing passive resistance and pile length.

Pile Length.—The author recommends that in determining pile length a factor of safety be introduced by using a fraction of the computed passive resistance. The length is then increased 20%—thus, a double factor of safety is introduced. When the problem involves clays and a factor of safety is used in the active pressure as well as in the passive pressure, then the extra 20% in length need not be considered.

Medium Clay.—The writer suggests that the term "medium" when applied to a clay be restricted to a description of the clay's plasticity. The term "firm" could be used to describe a clay having an unconfined compressive strength of 0.5 ton per sq ft to 1 ton per sq ft.

Increase of Shear Strength.—In Part I (under the heading, "The Shear Resistance of Soils and the Angle of Repose") it is stated that increased shear strength is harmful. This statement should be limited to the increased strength

[59] "Performance of Steel Sheet Piling Bulkhead," by Norman D. Lea, *Proceedings*, Third International Conference on Soil Mechanics and Foundation Eng., Zurich, Switzerland, Vol. II, 1953, p. 189.

caused by passive pressure at the toe. This effect will normally be small, whereas the increased strength resulting from a surcharge can be large and beneficial.

Anchor Connections.—Particularly in clay soils, there is a great reduction in the load on the anchor rod when yielding is allowed. This yielding may not occur until the yield point of the metal is reached; it is thus important that all connections be stronger than the rods.

S. PACKSHAW,[60] M. ASCE.—A valuable service has been rendered by the review of the design of anchored bulkheads and by the summary of the research results in this field.

In Part I (under the heading, "Unbalanced Water Pressure") attention is drawn to the influence of seepage forces in reducing the passive pressure coefficient K_P. Hitherto this effect has been almost completely ignored, as if no reduction existed. However, this omission is qualified by the fact that the value of H_u in most bulkheads is small in comparison to the depth of penetration D. Thus, the corresponding reduction $\Delta\gamma'$ in the unit weight of the retained soil (Fig. 5(*c*)) may therefore amount to only a few pounds per cubic foot—an insignificant amount. If H_u exceeds approximately 3 ft, most designers attempt to relieve the hydrostatic head by providing weep holes in the bulkhead and, if necessary, filters.

The influence of seepage forces, however, is too important to be neglected when allowance must be made for higher values of H_u, particularly in cofferdams and sheeted excavations. Examination of Mr. McNamee's work[61,62] shows that the coefficient, 3, in Eq. 3 is actually (as stated by Mr. Terzaghi) somewhat greater—approximately 3.7 for a ϕ-circle wedge of failure. The difference, although not large, may be of some consequence in the design of a cofferdam.

The author has made extensive comments on Mr. Rowe's research[19] on the relationship between the flexibility of the wall and the bending moment that it has to withstand. In a joint contribution[63] with J. Owen Lake, A.M. ASCE, to the discussion of Mr. Rowe's paper, the writer compared the results of Mr. Rowe's method with the conventional fixed-earth-support procedure. In the latter procedure, G_s is assumed to be equal to unity, but there is, in fact, a substantial factor of safety against ground failure because the penetration is designed to be sufficiently great to assure fixity. A similar comparison to the author's method of design may be of value.

Fig. 28 shows three typical bulkheads with retained heights of 20 ft, 30 ft, and 40 ft. Fig. 29(*a*) and Fig. 29(*b*) show the relationship between the flexibility number and the bending moment (or stress) for these bulkheads when retaining loose sand and dense sand, respectively. In Fig. 29, notation such as 4B-228 is used to signify a 4B piling section[64] stressed to 22.8 kips per sq in. The corresponding penetrations and tie-rod loads are given in Table 4.

---

[60] Director, The British Steel Piling Co., Ltd., London, England.

[61] "Seepage into a Sheeted Excavation," by J. McNamee, *Geotechnique*, Vol. I, 1949, p. 235, Fig. 4.

[62] *Ibid.*, p. 235, Fig. 5.

[63] Correspondence of J. Owen Lake and S. Packshaw of "Anchored Sheet-Pile Walls," by P. W. Rowe, *Proceedings*, Inst. of C. E., London, England, Vol. 1, Pt. 1, 1952, pp. 621–633.

[64] "Larssen Steel Sheet Piling," The British Steel Piling Co., Ltd., London.

In the conventional method of design, it is assumed that the wall will be fully fixed and that its deformation below the dredged level will be such that the passive pressures required for the formation of the necessary negative bending moment can be developed; there is no logical justification for this assumption. However, Mr. Rowe's research showed that full fixity (that is, a

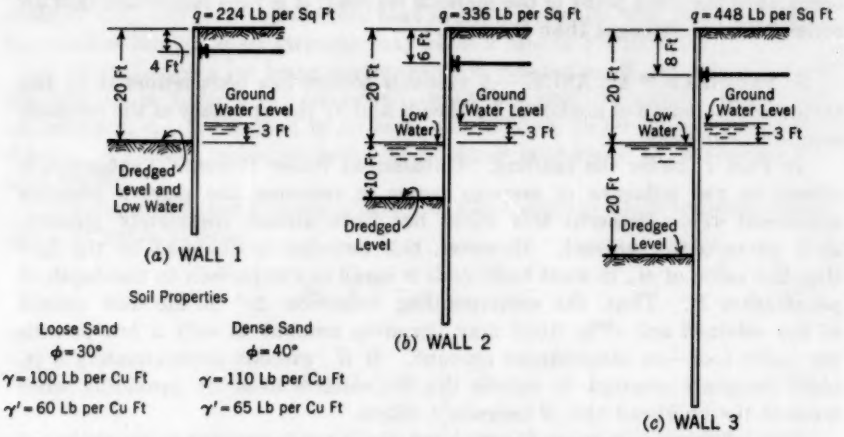


FIG. 28.—RETAINING WALLS USED FOR COMPARING DESIGN METHODS

negative bending moment almost as great as the positive) always develops when the wall penetrates dense soil. On the other hand, only a small degree of fixity is obtained with walls that penetrate loose soil although there is some negative moment, however slight, in every case. Realizing that loose, non-

TABLE 4.—THE RESULTING PENETRATIONS AND TIE-ROD LOADS FOR THE

Wall	(a) LOOSE SAND, $\phi = 30°$							
	PENETRATION, IN FEET				TIE-ROD LOAD, IN KIPS PER FOOT OF WALL			
	Conventional	K. Terzaghi[a]	P. W. Rowe	K. Terzaghi, allowing full flexibility reduction[a]	Conventional	K. Terzaghi	P. W. Rowe[b]	K. Terzaghi, allowing full flexibility reduction
1	15.5	16.8	14.8	16.8	4.7	5.8	4.6	5.8
2	22.5	22.6	22.0	22.6	10.3	13.0	9.9	13.0
3	28.4	28.8	25.3	28.8	18.1	22.0	18.1	22.0

[a] Including the recommended addition of 20%. [b] Corresponding to a

cohesive soils are rather uncommon in their natural state, it can be argued that most walls designed by the conventional method develop a major part of the required fixing moment. This statement will doubtless cause some satisfaction to those engineers who were compelled to use the conventional method prior to the publication of Mr. Rowe's research.

The conventional method of designing anchored bulkheads (which was introduced[65] in 1931 and has been used for the design of thousands of retaining walls throughout the world) deserves another comment: There have been no known cases of failure or excessive deflection which could be attributed directly to the method of design and not to the causes referred to by the author in

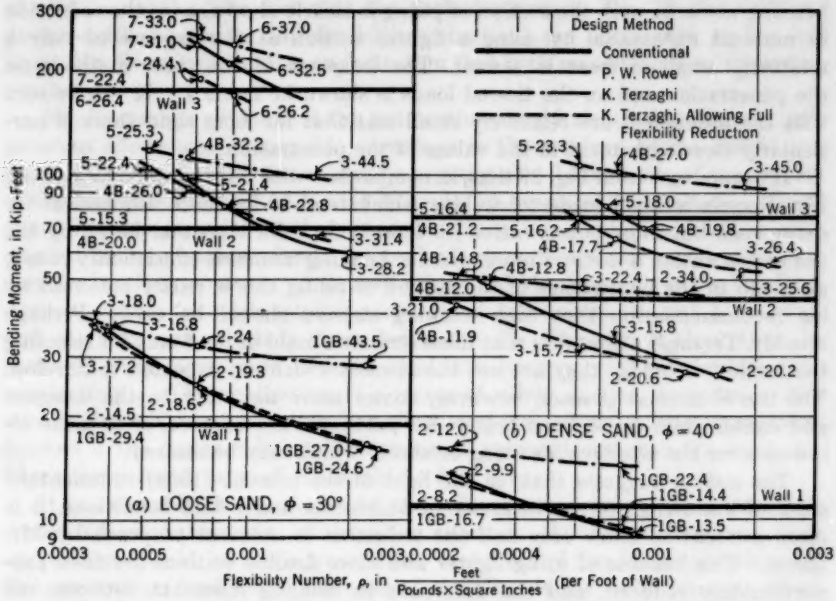

Bending Moment, in Kip-Feet
Flexibility Number, ρ, in Feet / Pounds × Square Inches (per Foot of Wall)
Design Method
Conventional
P. W. Rowe
K. Terzaghi
K. Terzaghi, Allowing Full Flexibility Reduction
(a) LOOSE SAND, φ = 30°
(b) DENSE SAND, φ = 40°
Wall 1
Wall 2
Wall 3

FIG. 29.—THE RELATIONSHIP BETWEEN BENDING MOMENT AND FLEXIBILITY NUMBER FOR THE RETAINING WALLS SHOWN IN FIG. 28

VARIOUS METHODS OF DESIGNING THE RETAINING WALLS SHOWN IN FIG. 28

(*b*) DENSE SAND, $\phi = 40°$								
PENETRATION, IN FEET				TIE-ROD LOAD, IN KIPS PER FOOT OF WALL				Wall
Conventional	K. Terzaghi[a]	P. W. Rowe	K. Terzaghi, allowing full flexibility reduction[a]	Conventional	K. Terzaghi	P. W. Rowe[b]	K. Terzaghi, allowing full flexibility reduction	
9.0	7.7	7.0	7.7	2.6	3.2	2.7	3.2	1
13.8	10.8	11.1	10.8	6.1	7.3	5.2	7.3	2
17.4	13.3	13.2	13.3	9.7	12.5	9.4	12.5	3

steel-pile section which will develop stresses within the prescribed limits.

Part II (under the heading, "Safety Requirements: Bulkhead Failures"). This leads to the general conclusion that the conventional method, when applied with proper consideration of the site conditions, is not unsafe.

[65] "Einspannungsverhältnisse bei Bohlwerken," by Hermann Blum, Wilhelm Ernst and Son, Berlin, Germany, 1931.

In Fig. 29(*a*) and Fig. 29(*b*) the results of the conventional method, which is unaffected by flexibility, are indicated by horizontal straight lines. The moments obtained by using Mr. Terzaghi's method were derived from one of the proprietary ranges of steel piling and are shown by the dash-dotted lines; those from Mr. Rowe's method are shown by solid lines. The variation in bending moment with the section of piling is clearly shown, as is the reduction in moment obtainable by using a lighter section at the expense of only a relatively small increase in stress. The influence of the three methods on the penetration and on the tie-rod loads is shown in Table 4. It can be seen that the differences are relatively small and that for loose sand there is particularly close agreement in the values of the penetration.

It is apparent from Fig. 29 that, in comparison with conventional procedure, Mr. Rowe's method leads to designs which tend to be more economical for dense soils but more conservative for loose soils. On the other hand, by the use of the author's method much greater bending moments consistently result, although in the final choice of the section of piling this is partly balanced by his recommendation that high working stresses should be used. Perhaps it is Mr. Terzaghi's intention that these stresses should be used only for selecting the section of piling; they are not the stresses which are expected to develop. The use of normal stresses, however, comes more naturally to the designer and enables him to be consistent in all parts of the structure, or to make allowance for the effective life (long or short) which may be desired.

The author suggests that, in the light of the present (1954) rudimentary state of knowledge of the behavior of bulkheads under field conditions, it is more prudent to allow only half the reduction in moment proposed by Mr. Rowe. The benefits of using lighter and more flexible sections are then proportionately reduced, and the difference in bending moments between one section and another often becomes so small that, within the limits of the range of available sections, it is not worth taking into account. To illustrate the effect of applying the full reduction proposed by Mr. Rowe to the author's method, curves shown by dashed lines have been included in Fig. 29. If these curves are used as a basis of design, it is suggested that more conservative stresses than those recommended by Mr. Terzaghi be used in the choice of a suitable section of piling.

Mr. Rowe has suggested[19] that design should be based on a factor of safety, G_s, of 1.5 in computing the passive pressure and that wall friction should be neglected. These recommendations actually result in G_s being at least 3. Mr. Rowe also introduces a horizontal shear force and a vertical end thrust at the bottom of the wall as secondary sources of moment reduction. One may doubt the existence of the end thrust or the importance of the shear force. In any case, these forces are small in comparison with the main cause of moment reduction—the flexibility of the wall—and may therefore be neglected without serious error. The reason for taking G_s equal to 1.5 and neglecting friction in determining the free-earth-support moment is that by doing so the computed moment can be made to agree with that which was actually measured in Mr. Rowe's tests on walls having less than the critical flexibility. A computation based directly on conventional earth-pressure theories, with

an allowance made for wall friction and with no factor of safety, results in bending moments that are too large to agree with the test results. Probably, for similar reasons, Mr. Terzaghi recommends that the free-earth-support moment should be derived from the coefficients of Table 2, which allow for wall friction, divided by a factor of safety of not less than 2 (except in the case of silts or clays). The fact that the dashed curves in Fig. 29 (the author's design method allowing full flexibility reduction) are fairly close to the full lines (Mr. Rowe's method) is an indication that both procedures lead to similar results.

Mr. Terzaghi's advice that the penetration should always be designed with generous allowances for possible variations in the retained height and in the assessment of soil properties cannot be questioned. Indeed, one of the virtues of the conventional method is that it results in generous penetrations. However, the recommendation that, in determining the passive pressure, a substantial factor of safety should be used affects not only the penetration but also the bending moment. A figure[66] offered by Mr. Rowe gives the bending-moment curves for walls retaining loose sand and driven (*a*) into loose sand and (*b*) into sand which is dense below 0.6 H. The curves show that, in changing from dense sand to loose sand, the bending moment doubles (approximately). The application of a factor of safety of 2 to the value of K_P selected from Table 2 has a similar effect, and this effect becomes even more marked if the designer has to contend with material such as silty sand, for which the reduced value of K_P might be 2 or less.

It is inevitable that a change in the soil properties below the dredge line should influence the design of the piling above it. It appears, however, that an attempt should be made to separate the effect of the factor of safety on the penetration (and hence the over-all stability of the wall) from its effect on the bending moment and stress in the sheet piling. A simple method of performing this separation would be to use a factor of safety equal to unity and to allow for wall friction in determining the penetration for the free-earth-support condition. The penetration should then be increased by a factor of 2 (or more) so as to make it correspond approximately to the penetration for a fixed wall. Then the full flexibility reduction could be applied to the free-earth-support moment and a section selected which can withstand that moment without exceeding the normal stresses for the material. This procedure, however, has the disadvantage that the free-earth-support moment computed on this basis is substantially greater than the value of M(max) (Figs. 13 and 16) corresponding to that obtained by actual measurement in Mr. Rowe's tests. Whatever procedure is adopted, much must be left to the discretion of the designer and to his interpretation of the soil and site conditions. The amount of the reduction in the bending moment that may be made in mixed sand or silt and clay strata must also be decided by the individual designer.

Mr. Rowe found that, notwithstanding the care taken in installing the apparatus, there were substantial variations (up to 50%) in the tie-rod loads. However, these variations are likely to be much smaller in an actual bulkhead

[66] "Anchored Sheet-Pile Walls," by P. W. Rowe, *Proceedings*, Inst. of C. E., London, England, Vol. 1, Pt. 1, 1952, p. 48, Fig. 16(a).

than in the model. The author suggests that no allowance should be made for the reduction in tie-rod load caused by the flexibility of the piling. Fig. 14 shows that for loose sand the reduction averages approximately 30% and that for dense sands it is greater than 50%. Although the tie rods and anchorages are only a minor part of the total cost of a bulkhead, the influence of flexibility seems too important to be neglected altogether. It is suggested, therefore, that (*a*) the tie-rod loads should be increased by 15% to 25% so as to allow for the differential yield of the tie rods and anchorages and (*b*) the full flexibility reduction should be permitted.

Mr. Terzaghi advises in Part II (under the heading, "General Design Procedure") that sand should be classified as silty if more than 5% passes the 200-mesh sieve. It would be instructive to have information concerning the line of demarcation between silty sand and silt, and the average values for K_A and K_P for (*a*) gravel or sand-and-gravel mixtures and (*b*) backfilling of broken rock.

In Eqs. 14 and 15 and in the corresponding pressure coefficients for silt and clay in Table 2, the author gives expressions which are applicable to smooth, frictionless walls. If adhesion is taken into account, the value of q_u can be multiplied by as much as 1.4 for the active pressure and 1.3 for the passive pressure[67] if the adhesion is equal to the shear strength of the clay. The effect of adhesion on the active pressure is relatively small, but it may be substantial on the passive pressure. This effect is in accordance with Mr. Terzaghi's recommendation in Part II (under the heading, "Safety Requirements: Passive Earth Pressure") of a somewhat lower value of G_s for silt and clay.

In Part II (under the heading, "Safety Requirements: Allowable Fiber Stresses in Sheet Piles") the author states that, if the piles have their locks on the neutral axis, the design should be based on the assumption that the locks are lubricated. Whether sections of piling of this type act as separate units, or whether the friction in the locks is sufficient to develop the full interlocked section modulus, has been discussed frequently and at length. It should also be realized that, although the piles behave as single sections when the locks are oiled, a sprinkling of dry sand will cause sufficient friction to develop approximately 75% of the interlocked modulus.[68] More friction, sufficient to develop almost the whole of the modulus, may be expected when the piles are driven into the ground. After the piles have been in position, even for a short time, the formation of rust inside the locks also causes them to act as solid joints.[69]

Mr. Terzaghi, in Part I (under the heading, "Influence of Flexural Rigidity on Bending Moment"), investigates the possibility of piles driven into clay behaving as "fixed" piles. The consolidation of the clay can be expected to cause a transition from a fixed-support condition to a free-support condition or to some intermediate stage. However, in the absence of any field observations it is impossible to state how long this process would take. It may be

[67] "Earth Pressure and Earth Resistance," by S. Packshaw, *Journal*, Inst. of C. E., London, England. February, 1946.

[68] "Die Spannungen in der Larssenwand," by E. Lohmeyer, *Die Bautechnik*, 1937, No. 53.

[69] "Civil Engineering Code of Practice No. 4 (1954)—Foundations," Pt. 4, Inst. of C. E., London, England, 1954, Item 4.33072.

that even firm clay may be stiff enough to permit several decades to elapse before a substantial change in the condition of support occurs. Innumerable bulkheads have been designed by the conventional method to be fixed in clays of varying grades, but there is no evidence that any have developed the noticeable deflections which one would associate with the change to free support.

There is one other aspect of bulkhead design which deserves comment—namely, the necessity of providing an anchorage which is not only of sufficient size but is also able to withstand its loads with a minimum amount of movement or consolidation. On this basis, an anchorage consisting of backward and forward raking piles forming an A-frame seems to be of doubtful value unless capped with a platform which, together with the overlying soil, imposes a vertical load on the piles. The value of ordinary anchor blocks which develop their resistance from the passive pressure of the ground in front of them may be much reduced if the ground against which they are to bear is disturbed during excavation for the blocks and tie rods. Particular care should then be taken in selecting the quality of the material for the backfilling and in ramming it against the bearing faces of the blocks.

J. VERDEYEN[70] AND V. ROISIN.[71]—Several of the concepts offered by Mr. Terzaghi are not agreed with by the writers. In the case of the forces acting on the inner face of a bulkhead, there is a linear distribution of active earth pressure (as shown in Fig. 15) when a bulkhead is loaded by soil placed behind the wall. The linear distribution also occurs in the case in which an excavation has been made along the face of a bulkhead which has an anchorage that yields so as to make the wall rotate about the foot. When the anchorage is nonyielding, however, the pressure distribution assumes that distribution shown in Fig. 30. The differences between Figs. 15 and 30 are quite important, as they result in theoretical loads on the anchorage that differ widely.

To illustrate the differences between the results obtained by use of the concepts offered in Figs. 15 and 30, the anchor pull A_p has been computed for Bulkheads A and B (shown in Fig. 31):

Bulkhead	A_p, in tons per linear yard	
	Fig. 15	Fig. 30
A	11	21
B	9.7	18

These differences appear to be quite large, and one might tend to discount the results obtained by the use of Fig. 30. Experimental investigation, however, has shown (Fig. 32(*a*)) that for Bulkhead A (with nonyielding supports) the value of A_p was even greater than 21 tons per lin yd.

Bulkheads A and B were used in 1953 on a project in Brussels, Belgium. The bulkhead wall and the props were made from DIN 22 and DIN 24 sections (Bulkhead B) and from DIN 32 sections (Bulkhead A). The properties of these sections are given in Table 5. The abscissa of Fig. 32 gives the time, in days, from the beginning of the project. At the end of the twenty-third day, the

[70] Cons. Engr. and Prof. of Civ. Eng., Brussels Univ., Brussels, Belgium.
[71] Assistant in Civ. Eng., Brussels Univ., Brussels, Belgium.

earth lying between Bulkhead A and the foot of the prop was removed until it was level with the lower face of the reinforced concrete floor on the left of Fig. 31(*a*). For Bulkhead B a period of 8 days was needed for a similar operation. Of the six type B bulkheads that were investigated, three (curves B9, B13, and B14 in Fig. 32(*b*)) had nonyielding supports, and three (curves B10, B11, and B12 in Fig. 31(*b*)) had supports that yielded $\frac{1}{8}$ in. The resulting

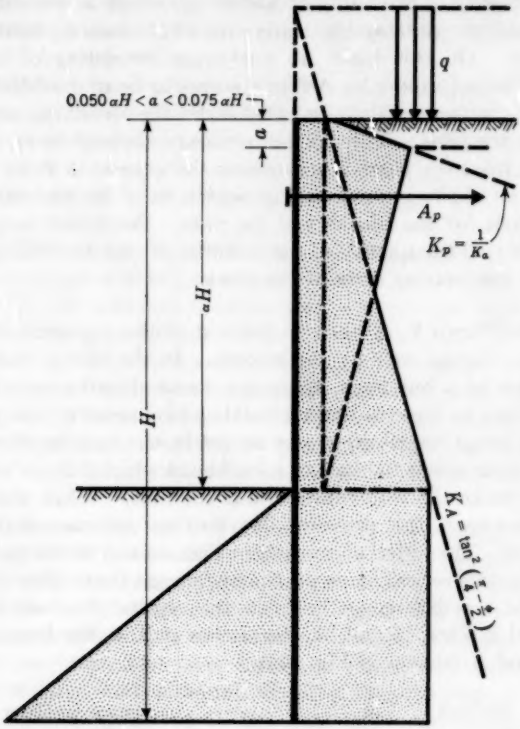


FIG. 30.—LATERAL EARTH PRESSURE ON A BULKHEAD HAVING A NONYIELDING ANCHORAGE

mean curves show that the anchor pull on bulkheads with nonyielding supports is considerably greater than the anchor pull on bulkheads with yielding supports.

It is of interest to note the considerable difference between the computed load and the in-place measurements. The differences are not surprising when one considers the lack of homogeneity of the retained earth (clay lentils and old foundations) and the lack of information about the mechanical properties of the soil. From an investigation of the neighboring soil, the following values were used in the theoretical determinations: $\phi = 35°$, $\delta = 0$, $c = 0$, $\gamma = 1.43$ tons per cu yd, and $\gamma' = 0.86$ tons per cu yd. Because of the lack of homogeneity, the computations may be considered to have an unsound basis. From the measurements, however, it can be seen that the deformability of the support greatly influences the anchor pull.

It is surprising that in Part II, under the heading, "Safety Requirements," Mr. Terzaghi makes no mention of the change in anchor pull with the passage of time. In the case of a backfilled bulkhead, it is believed prudent to use as

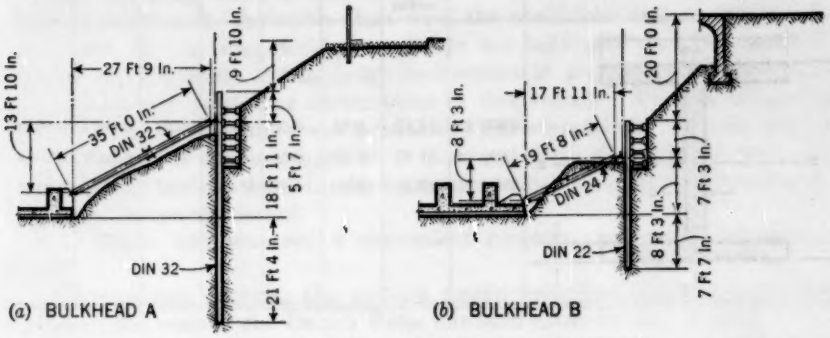


FIG. 31.—CROSS SECTIONS OF EXPERIMENTAL BULKHEADS

the maximum load that load which corresponds to the restoration of the at-rest earth pressure. This load is characterized, according to Curt Kollbrunner,[72] by

$$K_0 = 1 + \tan^2 \phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

Eq. 22 is applicable when the depth of embedment is less than a given depth, z_0, which is characteristic of the depth of the soil layers. Below this critical depth z_0, the pressure is assumed to be constant. Although Eq. 22 is applicable

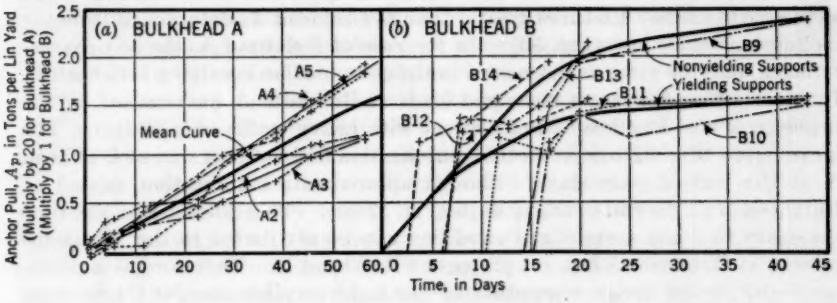
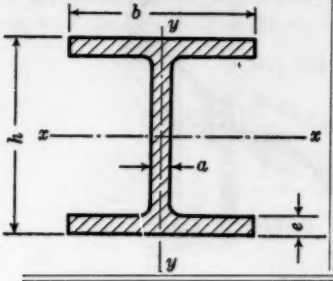


FIG. 32.—THE RELATIONSHIP BETWEEN ANCHOR PULL AND TIME FOR THE EXPERIMENTAL BULKHEADS

to loaded friction piles, it may be used for bulkheads because of the agreement between pressures computed from Eq. 22 and those pressures determined from an assumption of a linear distribution of pressure. In this manner, the safety of a sheet-pile anchored bulkhead decreases to a minimum value when the consolidation of the disturbed soil is complete. Therefore, different safety factors should be used for the time at which a bulkhead begins to act and for the time at which consolidation has occurred.

[72] "Fundation und Konsolidation," Vol. II, by Curt Kollbrunner, Schweizer Druck-und Verlagshaus, Zurich, Switzerland.

TABLE 5.—PROPERTIES OF BULKHEAD MEMBERS

Cross section	Section	Area, in square inches	h, in inches	b, in inches	a, in inches	e, in inches	I_x, in inches^4	I_y, in inches^4
	DIN 22	14.12	8.66	8.66	0.39	0.63	0.1934	0.0683
	DIN 24	17.25	9.45	9.45	0.43	0.71	0.2808	0.0996
	DIN 32	26.55	12.60	11.81	0.51	0.87	0.7748	0.2381

In the case of a bulkhead loaded as a result of an excavation along its face, the problem is similar—if the anchorage is deformable. If the support is nonyielding, however, the adoption of the modified pressure law proposed in Fig. 30 allows for a minimum safety against the restoration of the natural pressure.

In Part II, the author states (under the heading, "Safety Requirements: Allowable Stresses in Anchor Rods") "The anchor pull may also increase because of the repeated application and removal of heavy surcharges." This statement is acceptable; however, the effect of this phenomenon is minimized by Mr. Terzaghi. In fact, for Bulkhead A (Fig. 31(*a*)) measured A_p-values were approximately 1.5 times greater than the value of A_p determined from the modified pressure law (Fig. 30). In the case of Bulkhead A, the soil characteristics were the same as those used in the computation involving Bulkhead B. It appears that the larger measured loads at Bulkhead A were caused by the presence, above Bulkhead A, of a street with heavy traffic (Fig. 31(*a*)). The mean curve (Fig. 32(*a*)) shows that stabilization had not occurred at Bulkhead A at the end of sixty days, although approximate stabilization existed at Bulkhead B at the end of thirty days (Fig. 32(*b*)). This difference in the time necessary to reach a stabilized condition can be attributed to the vibrations present at Bulkhead A but not present at Bulkhead B. Therefore, it is necessary—during the design computations—to make an allowance for the increase of pressure caused by vibratory actions on the backfill, rather than introduce a correction to the safety factor at the conclusion of the computation.

Mr. Terzaghi has studied anchored bulkheads by performing a complete analysis of a bulkhead anchored at one level. The author implies that the method of investigation is also applicable to bulkheads anchored at several levels, but in this case a linear distribution of pressure does not exist. A more applicable and consistent representation of the pressure distribution is similar to that shown in Fig. 30.[73]

[73] "Nouvelle théorie du soutènement des excavations profondes," by J. Verdeyen and V. Roisin, *Annales*, L'Institut technique du Bâtiment et des Travaux Publics, Paris, France, June, 1952, No. 54, pp. 601–628.

KARL TERZAGHI,[74] HON. M. ASCE.—The well-documented case record offered by Mr. Wu is an instructive example of competent interpretation of the results of the subsoil exploration for the site of a dredged bulkhead.

The boring records indicated that the soil conditions west of Oregon St. were generally more favorable than were the conditions east of this street. Therefore, the designer decided to divide the bulkhead into two sections, meeting at Oregon St., and to design each section in accordance with the most unfavorable soil conditions encountered in this section. Table 3 illustrates the fact that this procedure was satisfactory and economical. In each section the scattering of the measured strut pressures from the average was considerable, but in both sections the maximum value was close to the value on which the design was based.

Mr. Bigler has proposed a convenient graphical procedure for solving Eq. 17.

A comparison between the writer's design procedure and the bulkhead design on the basis of the Danish Rules has been made by Mr. Hansen. The Danish Rules involve the assumption that the lateral pressure on the bulkhead increases from a minimum at about midheight of the bulkhead in an upward as well as in a downward direction. The experiments performed by Messrs. Tschebotarioff[18] and Rowe[19] indicate that such a pressure distribution develops only under special conditions. In practice, the existence of these conditions is rather uncommon; yet, according to Mr. Hansen, none of the bulkheads designed on the basis of the Danish Rules has failed. Hence, it can be concluded that the factor of safety for these bulkheads is considerably lower than the designers assumed—without being low enough to cause the bulkheads to fail. There is no doubt that the design of these bulkheads was economical; however, similar economies could be realized by reducing the factor of safety in design procedures which are based on more conservative assumptions concerning the distribution of the lateral pressure.

Also—according to Mr. Hansen—a comparatively rigid bulkhead (such as a bulkhead consisting of reinforced concrete sheet piles) has a greater reserve of strength than a flexible one because the flexible bulkhead has already exhausted the possibilities for a redistribution of the earth pressures, whereas the rigid one has scarcely begun its redistribution. Rigidity, however, is commonly associated with low deflection at failure, and a rigid bulkhead may fail before the deflection can produce a significant redistribution of the pressure. For the same reason, the struts in an open cut may fail by buckling before they have yielded sufficiently to induce a radical redistribution of pressure. Hence, the design of earth supports on the basis of the state of failure in the supported earth may lead in some, if not in many, cases to disappointing results.

Mr. Tschebotarioff's discussion is principally concerned with a comparison of his equivalent-beam method and Mr. Rowe's method of designing bulkheads composed of sheet piles driven into sand. Because this comparison is rather involved, one must begin with a brief review of the essentials of the subject (Fig. 22(*b*)).

---

[74] Prof. of the Practice of Civ. Eng., Harvard Univ., Cambridge, Mass.

In Fig. 22(*b*) the abscissa represents Mr. Rowe's flexibility number, ρ, which is a measure of the degree of flexibility of a sheet pile; the term ρ is defined in Eq. 12. The curves in Fig. 22(*b*) labeled "P. W. Rowe's Mean Curves" represent a digest of Mr. Rowe's test results. They show the relationship between flexibility number and moment reduction for dense, average, and loose sand.

From 1944 to 1948, Mr. Tschebotarioff performed tests[18] on bulkheads with various flexibility numbers. In Fig. 22(*b*) the range covered by these tests is labeled the "Range of Princeton Tests." It was found that the measured bending moments were approximately equal to those computed on the assumption that the upper part of the sheet piles is freely supported at the dredge line (Fig. 23(*b*)). This upper part was termed the "equivalent beam," and the procedure for computing the bending moments in the sheet piles is referred to in Fig. 22(*a*) as "G. P. Tschebotarioff's Equivalent Beam Theory." In Fig. 22(*b*) the moment reduction computed on the basis of this theory is represented by an horizontal line, labeled "G. P. Tschebotarioff's Design Proposal Line." The position of this line in Fig. 22(*b*) is independent of both the flexibility number and the density of the sand into which the piles were driven. Mr. Tschebotarioff qualified his recommendation by the following statement:

> "There is, however, no justification in applying the writer's proposal (Fig. 23(*b*)) indiscriminately to unusual conditions differing radically from those to which it referred * * *. Such misuse might indeed lead to results on the unsafe side.[42]"

However, before Mr. Rowe published his data[19] Mr. Tschebotarioff could not make any definite suggestions for the computation of the bending moments under "unusual conditions" because he had no observational data beyond the "Range of Princeton Tests" at his disposal. Furthermore, at that time he could not furnish any specific information concerning the limits for the validity of his theory because one of these limits was located in "unexplored territory," on the left side of the "Range of Princeton Tests" in Fig. 22(*b*).

Mr. Rowe supplemented Mr. Tschebotarioff's investigations by many more tests covering the entire range of conditions which can conceivably be encountered in designing bulkheads composed of sheet piles driven into sand. Furthermore, Mr. Rowe introduced into the investigations the concept of the flexibility number; he divided his test data into groups, according to flexibility number and relative density of the sand below the dredge line (all of which had not been done before). Mr. Rowe devised the graphical method of plotting his classified test results for which Fig. 22(*b*) is an example. The test results themselves are represented by the curves labeled "P. W. Rowe's Mean Curves." These curves illustrate the fact that the moment reduction depends on both the flexibility number and on the density of the sand, whereas Mr. Tschebotarioff's reduction factor is independent of these parameters.

At the present time (1954) "P. W. Rowe's Mean Curves" in Fig. 22(*b*) represent the only classified set of experimental data concerning the influence of flexibility and sand density on moment reduction. Therefore, these curves have been used by the writer as a basis for estimating the errors associated with Mr. Tschebotarioff's simplified procedure for computing the reduction factor.

The term "error on the unsafe side" indicates an excess of Mr. Rowe's reduction factor over Mr. Tschebotarioff's reduction factor, in percentage of the latter. Hence, if

$$r_T = \frac{M}{M(\max)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (23)$$

is the reduction factor computed by Mr. Tschebotarioff's theory for sheet piles with a given flexibility number ρ and if r_R is the reduction factor determined by the ordinate of Mr. Rowe's curve for loose sand—for sheet piles with the same flexibility number—the error is defined by

$$e_{RT} = 100 \frac{r_R - r_T}{r_T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

From Fig. 22(*b*) the following information can be obtained concerning the magnitude of these errors. Within the "Range of Princeton Tests" the error does not exceed 14%. Hence, within this range, Mr. Tschebotarioff's procedure represents an expedient and reliable method for computing the moment reduction. According to Fig. 22(*b*) the legitimate use of this procedure is limited to sheet piles with a flexibility number of more than approximately 18 in.^{2} per lb per ft—that is, within this range the designer can safely use the procedure of either Mr. Tschebotarioff or Mr. Rowe. However, at the left boundary of Mr. Skempton's "Normal Range," corresponding to a flexibility number of approximately 12 in.^{2} per ft and $\log \rho = 3.3$, the error on the unsafe side is 66%, which is excessive. Hence, if the flexibility number of the sheet piles is less than approximately 18 ($\log \rho = -3.0$), the moment reduction should be determined only by Mr. Rowe's curves.

Because Mr. Tschebotarioff's method of design is expedient, the following procedure is indicated: The sheet piles should be dimensioned by the equivalent-beam method. If the flexibility number is beyond the "Range of Princeton Tests," it is necessary to repeat the computation—this time by trial and error on the basis of Mr. Rowe's mean curves in Fig. 22(*b*), as described under the heading, "Computation of Maximum Bending Moments."

Mr. Rowe's procedure too cannot be considered perfect because tests conducted on a larger scale and under different conditions may furnish flexibility-moment reduction curves which are somewhat different from those he obtained. At the present time (1954), however, his data are the most reliable and accurate enough for all practical purposes.

It is agreed that, as Mr. Tschebotarioff recommends, the dimensions assigned by Mr. Rowe to the flexibility number should be replaced by square inches per pound per foot. The writer retained Mr. Rowe's dimensional equation for the purpose of facilitating comparison with Mr. Rowe's original data. The conversion can readily be performed by using the curve in Fig. 22(*a*).

In order to explain his procedure, Mr. Tschebotarioff described (under the heading, "Example") the design of a bulkhead 45 ft high, backfilled with sand. According to the equivalent-beam theory, the reduction factor r_T is 0.45, and the bending moments make necessary the use of an MZ-22 section, represented in Fig. 22(*b*) by the MZ-22 curve, with a value for $\log \rho$ of approximately -2.85.

If the sand into which the piles are driven were loose, Mr. Rowe's curve for loose sand would necessitate a reduction factor r_R of 0.54 for the same sheet piles. Hence, in accordance with Eq. 24, the error, e_{RT}, involved in using Mr. Tschebotarioff's procedure would be 20%, a tolerable error.

The bulkhead which Mr. Tschebotarioff used as an example was assumed to retain a backfill consisting of sand. If, however, the same bulkhead were backfilled with dredged silt or clay—which is not an unusual condition—curves 1 and 2 in Fig. 22(*a*) and the corresponding curves marked MZ 22 to MZ 38 in Fig. 22(*b*) would be displaced to the left, because much heavier sheet piles with a lower flexibility number would be required to support the backfill. As a consequence, r_R is much greater than 0.54, and the error becomes important.

According to Mr. Tschebotarioff, both Mr. Rowe and the writer overemphasize the influence of the relative density of the sand below the dredge line and underestimate the effect of the depth of sheet-pile penetration on the conditions of end support. To illustrate his argument concerning the influence of relative density on moment reduction, Fig. 25 was presented. The position of points 54, 54A, and 56 in Fig. 25(*b*) is abnormal indeed. However, in tests 54 and 54A the backfill carried a heavy surcharge, and in test 56 the depth of sheet-pile penetration had the abnormally small value of 0.27 *H*. These facts probably explain why Mr. Rowe did not show these points in his diagram from which Fig. 25(*a*) is derived.

In the writer's opinion the influence of relative density on the conditions of end support can hardly be overemphasized. This opinion is based chiefly on experiences in pile driving and on the results of loading tests. If a sand stratum is dense, piles meet refusal at a depth of a few feet below the surface, whereas, if the sand is loose, the piles can be driven without difficulty to a depth of several tens of feet. The differences in resistance to pile penetration are associated with equally important differences in compressibility, which determine the value of the "coefficient of horizontal subgrade reaction." This fact is shown by the differences in the settlement of bearing plates on dense and loose sand at equal load and loaded area.

The driving of a single row of sheet piles cannot be expected to convert a loose sand into a dense one. Therefore, it is maintained that the influence of the relative density of the sand below the dredge line on the conditions of end support is at least as important as the influence disclosed by Mr. Rowe's test results.[19] For the same reason, the writer considers it necessary to adapt the depth of sheet-pile penetration to the relative density of the sand, assigning a lower K_p-value to the loose sand than to the dense sand. The writer does not concur with Mr. Tschebotarioff's proposal to drive the sheet piles to a depth of 0.43 *H* irrespective of the density of the sand.

Mr. Tschebotarioff further discusses the criteria for active and passive earth-pressure conditions. In connection with retaining walls and arch abutments, the terms "active" and "passive" earth pressure have a well-defined meaning. At the lower end of the sheet piles of an anchored bulkhead the entire body of sand in contact with the sheet piles moves out, and the sheet piles move with reference to the sand. Under such conditions the terms "active" and "passive" become meaningless abstractions.

Mr. Tschebotarioff mentions methods of reducing the lateral pressure by the use of sand fills. This procedure was used successfully by the writer in 1941 with a cellular bulkhead.[75] Mr. Tschebotarioff's subsequent experimental investigations of this procedure (Fig. 11, tests 4 and 5) led to the same conclusions as did the preceding observations on a full-sized structure. The principle is simple: The lateral pressure on the bulkhead depends on the material in which the potential surface of sliding is located. Hence, by replacing an unstable material, such as soft silt, by a granular material having a high angle of internal friction, the pressure on the lateral support is reduced.

Mr. Baumann is correct in stating that stresses in a flexible bulkhead can be accurately computed only by treating the bulkhead as a statically indeterminate system. However, the basic equations contain the coefficient of horizontal subgrade reaction, and the observational data required for estimating this value are inadequate (as of 1954).

A comment is also made by Mr. Baumann concerning the allowable stresses in the steel. In this connection, distinction must be made between the bending stresses in the sheet piles and the tensile stresses in the anchor rods. In the sheet piles the yield point is reached only in those short sections where the bending moment is close to the maximum. Because of this fact the plastic deformation of the steel is localized, and the energy required to produce a large deflection of the sheet piles is relatively small. In the anchor rods, however, the yield point is reached almost simultaneously at every point between anchor and wale, and the rapid elongation which ensues when the steel is stressed beyond the yield point is inconsequential, provided the connections are stronger than the rod. In some cases it is even beneficial.

Both Mr. Baumann and Mr. Packshaw discuss the writer's recommendation that locks located in the neutral plane of a row of sheet piles should be considered frictionless. The friction which develops in the locks during the process of driving the sheet piles depends not only on the type of soil which is penetrated but also on the design of the locks. The locks of some types of sheet piles are fairly tight whereas others are loose. The importance of the influence of lock friction on the flexibility number and, as a consequence, on the allowable moment reduction is strikingly revealed by the two curves labeled "Larssen V (Lubricated Joints)" and "Larssen V (Solid Joints)" in Fig. 22(*a*). However, a reliable estimate of the lock friction is impracticable. Therefore, one should design on the basis of the more unfavorable assumption, which is to consider the locks to be frictionless.

It is contended by Mr. Packshaw that "* * * the conventional method, [of bulkhead design], when applied with proper consideration of the site conditions, is not unsafe." However, in some cases it may be somewhat uneconomical. If, for example, the sheet piles are to be driven into dense sand, the conventional method necessitates an excessive depth of sheet-pile penetration, as shown by the values in Table 4(*b*).

---

[75] "Shipways with Cellular Walls on a Marl Foundation," by M. M. FitzHugh, J. S. Miller, and Karl Terzaghi, *Transactions*, ASCE, Vol. 112, 1947, pp. 298–324.

With respect to Fig. 29, Mr. Packshaw states:

"Perhaps it is Mr. Terzaghi's intention that these stresses should be used only for selecting the section of piling; they are not the stresses which are expected to develop."

This interpretation is perfectly correct. In order to explain his intention, the writer refers to the case record illustrated by Figs. 17, 18, and 19. The distance between the borings ranges between 300 ft and 500 ft. The soil conditions prevailing between the borings can only be approximated on the basis of the geological characteristics of the site; this procedure leaves a wide margin for error. The elevated position of the writer's curves in Fig. 29 with reference to the corresponding curves proposed by Mr. Rowe is intended to compensate for the inevitable errors involved in the interpretation of the boring records.

Mr. Packshaw also comments on the writer's proposal to tolerate not more than one half of the moment reduction determined by Mr. Rowe's curves. The writer himself was not quite satisfied with this proposal; therefore, it was decided to replace it by the following one, which appears to be more appropriate and, at the same time, is equivalent to the one suggested by Mr. Packshaw. If the boring records indicate that the sand is dense, the moment reduction should be computed by the use of Mr. Rowe's curve labeled "Medium" in Fig. 13, and, if they indicate a sand of medium density, the "Loose" curve should be used.

An attempt "to separate the effect of the factor of safety on the penetration (and hence the over-all stability of the wall) from its effect on the bending moment and stress in the sheet piling" is proposed by Mr. Packshaw. During the preparation of his paper the writer made several attempts of this kind; none of them led to a satisfactory result. Unless the subsoil conditions are very simple—and they seldom are—the designer is compelled to use his own judgment. No rules of general validity can be established.

Mr. Packshaw suggests that flexibility reduction be permitted not only for the bending moments in the sheet piles but also for the stresses in the anchor rods. The writer had originally intended to do this, but afterward he decided against it because of the uncertainties involved in the computation of the anchor pull.

The computation for the anchor pull is based on the assumption that the distribution of the active earth pressure on the bulkhead is in accordance with the classical earth-pressure theory. However, because of the rigidity of the anchorage (or of some other causes), arching may develop in vertical planes, whereupon the anchor pull is greatly increased. The performance of the bulkhead shown in Fig. 33 is an example of this phenomenon. The sheet piles were driven into dense sand, and the backfill consisted of clean sand compacted in layers. The pull on the anchor rods was computed without any flexibility reduction on the assumption that the angle of internal friction of the sand was 34°, which is a very conservative value for compacted sand. The tie rods were anchored to a reinforced concrete wall, forming one of the lateral boundaries of the backfill; therefore, the anchorage was very rigid. This condition is likely to increase the anchor pull considerably. Therefore, the allowable stress in the anchor rods was assigned the low value of 17 kips per sq in. Furthermore,

it was specified that the anchor rods should be installed after the backfill was raised to the level of the rods and that the connections should be considerably stronger than the rods.

During construction, the elongation of the tie rods was measured at various stages of the backfilling operations by determining the change of the distance between the outer ends of the anchor rods and those of slender dummy rods ("stretch rods"), which were attached to the anchor rods without being subjected to tension. On the basis of the results of these measurements, the stress

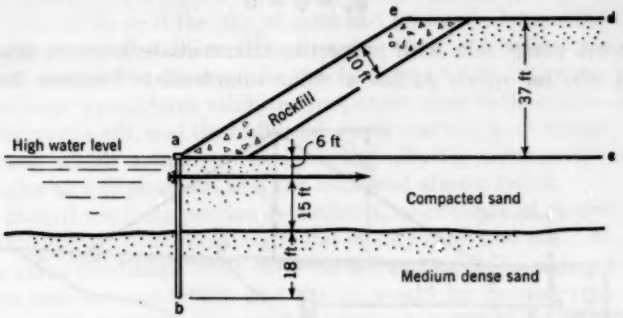


FIG. 33.—ANCHORED BULKHEAD IN DENSE SAND WITH SAND SURCHARGE AND BACKFILL

in the anchor rods was computed. After the filling operations were completed, the stresses in the anchor rods were 26 kips per sq in. Hence, the stresses exceeded the design stress by approximately 50%.

Mr. Packshaw inquires about the writer's conceptions concerning the line of demarcation between silty sand and silt. This most important question can only be answered by considering the angle of shearing resistance which has been assigned to these two materials. The increase Δs in the shearing resistance of a soil caused by an increase, Δp, of the pressure per unit of area on a potential surface of sliding is approximately equal to

$$\Delta s = (\Delta p - \Delta u) \tan \phi_s = \Delta p \tan \phi_a \quad \ldots\ldots\ldots\ldots\ldots (25)$$

In Eq. 25 ϕ_s is the "true" angle of internal friction obtained by a consolidated-slow shear test or triaxial test; Δu is the excess hydrostatic pressure in the pore water of the soil associated with the total increase Δp of the unit pressure; and ϕ_a is the apparent angle of internal friction, also termed the angle of shearing resistance. The excess hydrostatic pressure Δu is caused by the time lag between the increase of the total pressure by Δp and the corresponding decrease of the water content of the soil.

For silty sand, the writer assumed that the excess water drains out of the soil almost as rapidly as the total pressure is increased. On this assumption $\Delta u = 0$,

$$\Delta s = \Delta p \tan \phi_s = \Delta p \tan \phi_a \quad \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

and

$$\phi_a = \phi_s = \phi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27)$$

The angle ϕ has been assigned values intermediate between 34° and 26°, depending on the relative density of the silty sand.

For silt it was assumed (as it was for clay) that the excess hydrostatic pressure is equal to the increase Δp of the total pressure ($\Delta u = \Delta p$) and that it retains its value until at least the end of the period of construction, when

$$\Delta s = (\Delta p - \Delta p) \tan \phi_s \quad \ldots\ldots (28)$$

and

$$\phi_a = \phi = 0 \quad \ldots\ldots (29)$$

There are many soils with properties intermediate between those of silty sand and silt, for which ϕ_a has a value intermediate between 26° and 0°.

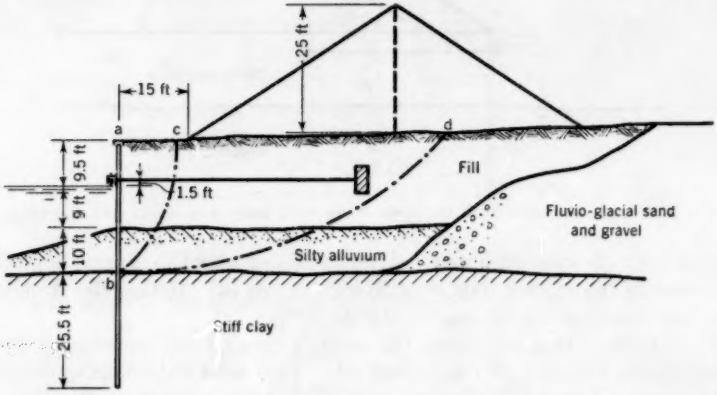


FIG. 34.—VERTICAL SECTION THROUGH AN ANCHORED BULKHEAD (BACKFILL OF SILTY SAND ON ALLUVIAL SILT)

According to the theory of consolidation, the value of ϕ_a for these soils depends on (*a*) the coefficient of consolidation of the soil, (*b*) the dimensions of the body of soil subject to an increase of pressure by Δp, and (*c*) the rate at which the pressure increases. The influence of the last two factors on the value of ϕ_a is illustrated in Fig. 34, representing a vertical section through an anchored bulkhead.

The sheet piles were driven through a layer of alluvial silt and fine silty sand to a depth of not less than 26 ft into stiff, glacial clay. The backfill consisted of silty sand. The deformations caused by the lateral pressure were imperceptible. Four years after the bulkhead was completed, a pile of sand with a diameter of 80 ft and a height of 25 ft was deposited within a few days on the surface of the backfill. According to Coulomb's theory the increase of the lateral pressure on the bulkhead caused by the weight of the sand should have amounted to less than 10% of the pressure exerted by the backfill. However, while the sand was being deposited, the upper edge of the bulkhead located next

to the foot of the sand pile moved out 9 in., whereupon the surface of the backfill was no longer used as a storage space.

The excessive outward movement of the bulkhead was caused by the rapid application of the surcharge combined with the relatively low permeability of the alluvium. The rapid application of the surcharge produced an excess hydrostatic pressure in this material, whereupon the potential surface of sliding moved from position bc into position bd, and the shearing resistance along bd was inadequate.

If the thickness of the layer of silty alluvium at the base of the fill had been 2 ft rather than 10 ft, or if the pile of sand had gradually been built up in the course of several months rather than in a few days, the excess water would have drained out of the layer while the load was being increased. Consequently, the layer would have exhibited properties close to those of a silty sand and not to those of a silt, and the bulkhead would scarcely have moved. Under the conditions which actually prevailed, the alluvial layer performed in a manner similar to a typical silt, and the bulkhead almost failed.

On the basis of the facts previously outlined, no criterion of general validity can be established for discriminating between silty sand and silt. In order to learn under given conditions what value for ϕ could safely be assigned to a soil intermediate between sandy silt and silt, it would be necessary to perform consolidation and triaxial tests on representative samples of the soil in contact with the bulkhead. An estimate must then be made of the excess hydrostatic pressures which are likely to develop in the soil during the period of construction.

The instances in which such elaborate investigations would be justified economically are rather rare. Under normal conditions, the engineer has the choice of guessing as to what value of ϕ_a can be assigned safely to the soil (at the risk of overestimating this value) or proceeding on the pessimistic assumption that the "intermediate" material is a silt ($\phi_a = 0$).

Mr. Packshaw inquires concerning the values which should be assigned to gravel and to rock fill. For loose gravel the writer would use the values proposed for medium sand, and for dense gravel, those for dense sand—or even less conservative values. Rock fill is invariably deposited by dumping, and the resulting fill is relatively loose. For dumped rock fill the values assigned to medium sand appear to be appropriate.

The writer agrees with Mr. Packshaw that the possibility of the fixed earth support of sheet piles driven into firm clay or stiff silt gradually changing into a free earth support is rather remote. If the sheet piles are embedded in soft silt or soft clay, such transformation is irrelevant because in these cases the bulkheads have to be designed on the basis of the assumption of free earth support without moment reduction.

The writer shares Mr. Packshaw's aversion to anchorages consisting of piles forming an A-frame, unless the frames are capped with a platform carrying the weight of the superimposed fill. If a pile is acted upon by a force tending to pull it out of the ground, its resistance against pulling decreases rapidly with increasing upward displacement. Therefore, the tension piles of A-frames

which do not carry any load may fail while the anchor rods are still intact. The writer has witnessed a failure of this type.

If the anchor pull is carried by anchor beams or anchor plates, the importance of thoroughly compacting the earth on the pressure side of the anchor cannot possibly be overemphasized. The writer appreciates that Mr. Packshaw has called attention to this fact.

The problem of computing the lateral pressure caused by irregular surcharges has been noted by Mr. Lea. He recommends that this pressure should be computed on the basis of Coulomb's theory. According to this theory, only that part of an irregular surcharge should produce an increase of the horizontal pressure on a lateral support which is located between the upper edge of the support and that of the potential surface of sliding.

In Fig. 35, ab represents an anchored bulkhead backfilled with sand; ac is the potential surface of sliding; and Q is a heavy, concentrated load located beyond ac. According to Coulomb's theory the influence of this load on the lateral pressure should be nil; yet there is no doubt that the application of this load is associated with an horizontal stretching of the underlying backfill,

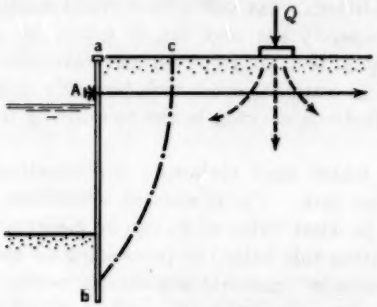


FIG. 35.—ANCHORED BULKHEAD WITH SAND BACKFILL SUPPORTING A LOAD, Q

involving an outward movement of the entire wedge, acb, whereas the position of the anchor point A remains practically unaltered. As a consequence of this action, the lateral pressure on the bulkhead increases. Repeated removal and reapplication of the load would cause a further increase of this pressure.

In Fig. 33 the body of sand, acde, represents a surcharge. According to Coulomb's theory, the lateral pressure exerted by the backfill and the surcharge combined should have produced a stress in the anchor rods of approximately 17 kips per sq in. Because of the rigidity of the anchorage the actual stress was approximately 26 kips per sq in. The writer anticipated this condition, however, and the precautions previously cited were taken.

Considering these and various other facts, Mr. Lea's recommendation that the lateral pressure caused by irregular surcharges should be computed on the basis of Coulomb's theory does not appear to be a step in the right direction.

Coulomb's theory and the method proposed by the writer furnish the limits between which the real values may range. In some cases the designer may feel justified in determining the lateral pressure by Coulomb's theory, but this should not be established as a general rule. The description of bulkhead performance shown in Fig. 34 indicates the great variety of factors on which the lateral pressure caused by irregular surcharges depends.

Mr. Lea proposes that a factor of safety of 2 be introduced into the equations for K_A and K_P in Table 2, replacing these equations by Eqs. 21*a* and 21*b*. The value of q_u is usually determined by unconfined compression tests on seamless-tube samples, and these values are somewhat lower than the q_u-value of the undisturbed material. Furthermore, the adhesion between the soil and the sheet piles has been disregarded. It is believed, therefore, that the values defined by the equations in Table 2 are conservative, provided that they have been selected on the basis of the most unfavorable results of the soil tests and not on the basis of averages.

Mr. Lea implies that the writer determined "* * * the passive resistance and pile length * * * for a factor of safety of unity." In Part II (under the heading, "Computation of Depth of Sheet-Pile Penetration") the writer specified that the depth of sheet-pile penetration should be computed for a factor of safety G_s and under the subsequent heading, "Safety Requirements," values were assigned to G_s, ranging from 1.5 to 2. The additional 20% increase of the depth of sheet-pile penetration is not intended to increase the factor of safety. It is proposed for the purpose of compensating for the errors involved in the interpretation of the boring records. As the spacing between borings is commonly more than 100 ft, important errors are almost inevitable.

Mr. Lea also notes that the anchor connections should be stronger than the anchor rods. Departure from this rule constitutes faulty design.

Redistribution of the active earth pressure may occur on dredged bulkheads as well as on filled bulkheads, according to Mr. Briske. This statement is perfectly correct. At the same time it must be remembered that it is—and always will be—impracticable to determine, with certainty, in advance of construction, what type of earth-pressure distribution will ensue. Therefore, sound engineering practice requires that design be based on the more unfavorable possibility.

In order to establish a favorable redistribution of the lateral pressure, Mr. Briske recommends prestressing the anchor rods. Under no circumstances would the writer advocate such a procedure because under field conditions this procedure may increase both the stresses in the anchor rods and the bending moments in the sheet piles.

Messrs. Verdeyen and Roisin present the results of their observations on two recently constructed sheet-pile bulkheads (Fig. 31). These bulkheads, like the one shown in Fig. 34, support a surcharge consisting of a thick layer of sand with a steep slope descending to the upper edge of the sheet piles. The lateral pressure caused by such a surcharge is of the same order of magnitude as that resulting from a backfill. It may have any value between the Boussinesq

value and the Coulomb value, depending on the yield of the anchorage. This fact alone eliminates the possibility of an accurate computation of the pressure. However, a slight yield of the anchor is sufficient to reduce the pressure to a value close to the minimum. This conclusion is corroborated by the observations on the bulkhead shown in Fig. 34. The Boussinesq pressure on this bulkhead exceeds the Coulomb pressure by several hundred percent and the yield of the anchor line was less than two tenths of one percent of the height of the bulkhead. However, the measured anchor pull exceeded the Coulomb value by not more than 50%.

A comparison between the measured and the Coulomb value of the earth pressure on bulkheads A and B in Fig. 31 cannot be made because no adequate data concerning the physical properties of the soils in contact with these bulkheads are available. These properties were unusual indeed because the lateral pressure on both bulkheads increased after completion by approximately 100%, whereas the pressure on the bulkhead in Fig. 33 became sensibly constant after the filling operations were completed.

In this connection, it was learned (by letter from Mr. Roisin) that the surcharge (the body of earth located above the level of the upper edge of the sheet piles) consists chiefly of coarse sand which was deposited at its present site in approximately 1880. The sand fill rests on fine, silty sand with layers of soft silt and clay ("sable ypresien") which is unstable in a saturated condition. The boundary between this material and the superimposed fill is located at the site of bulkhead A approximately at the elevation of the upper edge of the bulkhead and at bulkhead B approximately 15 ft below this edge. These conditions account for the extraordinary difference between the pressure per yard of length of the lateral supports (40 tons per yd at A and 2.5 tons per yd at B). However, judging from the design of the supports, such an important difference has not been anticipated.

It must also be considered that the water table was located approximately 10 ft above final grade of the excavation on the free side of the bulkheads. When the water table was reached, drainage wells were installed on the bottom of the braced trenches shown on the right side of the upper part of the sheet piles in Fig. 31, and the water table was lowered by pumping. The abnormal increase of the lateral pressure on the bulkhead after completed excavation operations can be accounted for by movements in the unstable ground, associated with the process of drainage by pumping and probably assisted by traffic vibrations.

Messrs. Verdeyen and Roisin ascribe the gradual increase of the lateral pressure exclusively to traffic vibrations and suggest that the coefficient of active earth pressure K_A of a backfill with an angle of internal friction ϕ may gradually assume a value

$$K'_A = K_0 = 1 + \tan^2 \phi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

There are many old retaining walls, both in the United States and elsewhere, supporting the foot of railroad fills on lines subject to heavy traffic. None of

them was designed with special allowance for the effects of traffic vibrations. If K'_A were one third of the value determined by Eq. 30, all these walls would have failed long since. The failures which have come to the writer's attention were a result of unfavorable foundation conditions, gradual or rapid softening of cohesive backfills, or formation of ice layers behind the back of the wall; none could be attributed to vibration effects.

It is noted by Messrs. Verdeyen and Roisin that the writer has not adequately considered the effects of repeated application and removal of surcharges on the lateral pressure. Every loading cycle produces a permanent increase of the lateral pressure. However, with an increasing number of cycles, the supplementary pressure produced by the cycle decreases and finally becomes zero. According to the writer's experience, the total increase is still well within the margin of safety, provided the lateral pressure caused by the surcharge has been computed by one of the procedures described in the paper.

Fig. 30 illustrates the proposals of Messrs. Verdeyen and Roisin concerning the assumptions which should be made about the distribution of the active earth pressure on the inner face of anchored bulkheads; this proposal appears to be over-conservative. The bulkhead shown in Fig. 18 is an example of the adequacy of the conventional assumption shown in Fig. 20. There are few anchored bulkheads with an anchorage as unyielding as the steel struts shown in Fig. 18; yet all the measured strut loads were smaller than those computed in the conventional manner.

The writer has reconsidered the recommendation in Part II (under the heading, "Computation of Maximum Bending Moments: Moment Reduction Resulting from Flexibility Effect"):

> "However, because of the rudimentary state of present (1953) knowledge of the performance of bulkheads under field conditions, the computed bending moment $M(\max)$ should be reduced by not more than $\frac{1}{2}(M(\max) - M_a)$."

This recommendation should be changed to read as follows:

> In order to compensate for the uncertainties involved in the assessment of the subsoil conditions between drill holes, the following procedure is recommended: If the test borings indicate that the sand below the dredge line is dense, the moments should be reduced in accordance with Mr. Rowe's "Average Curve." Otherwise, Mr. Rowe's curve for loose sand should be used.

The writer wishes to express his gratitude to the discussers for their valuable contributions. Their comments brought the controversial features of the topic clearly into prominence. Case records such as those illustrated by Figs. 18 and 31 demonstrate the practical value of careful observations on full-size bulkheads. Phenomena of the type described by Messrs. Verdeyen and Roisin are beyond the scope of theoretical forecast, and their existence can only be discovered by observations in the field.

In the last analysis, a paper dealing with anchored bulkheads does not serve its purpose unless it stimulates the thought and imagination of designers and makes them conscious of the consequences of the inevitable errors involved in the interpretation of the boring records and test results. Because of the great variety of subsoil conditions which may be encountered, the subject does, and always will, leave a wide margin for judgment—and also for misjudgment.

Discussion of
"THE SUCTION FORCE IN SOILS UPON FREEZING"

by A. R. Jumikis, M. ASCE
(Proc. Sep. 445)

D. P. KRYNINE, * M. ASCE.—An upward flow of moisture is a movement against gravity under the action of an agency stronger than gravitation. In general, this is suction caused by the substance in which or towards which the moisture flows. Another convenient term would be "attraction" of the moisture by that substance.

That the so-called capillary movement is caused by suction was established in 1935 by Schofield, who replaced the concept of the capillary potential by that of pF or decimal logarithm of the free energy of soil as expressed in cm. of water column.[1] D. J. McLean and his co-workers in England, as mentioned by Professor Jumikis, work primarily on the basis of the Schofield views. The measurement of soil suction as done by the highway engineers in England is discussed for general use in a very good text-book.[2]

In the so-called capillary motion of a liquid, the meniscus and the pressure deficiency in the fluid under the meniscus are secondary or subordinate phenomena, the real cause of the capillary movement being the action of the free energy of the unsaturated soil. The writer believes that the energy concept should be extended also to the water movement upon freezing in recognition of the fact that Professor Jumikis' sub-pressure is a subordinate phenomenon analogous to that existing in the common capillary motion, but of course, not identical to it.

As to the proposal by Professor Jumikis to consider the "suction force" as a soil property, the writer does not feel that this proposal should be accepted. In the first place, a unit pressure, positive or negative, is not "force" in the commonly accepted sense of the word; and secondly, such crude conventional devices as forces should not be used in considering phenomena like upward moisture flows, which are basically controlled by energy interchanges. And if suction is developed by some substance other than soil, should it be considered as the property of that particular substance?

Professor Jumikis has entered a vast realm of suction as produced by matter, and soils are only an example thereof. Plants absorb cool water from the soil and drive it in the form of sap up to warmer regions, and sometimes to incredible heights. The Grizzly Giant of the California Redwoods is 209 feet high and has practiced the cool water pumping to that height for about 3,900 years. The writer is by no means an expert in the explanation of these phenomena, but so far as he has been informed, the leaves (and probably the needles) of a tree produce a heavy suction that permits it to maintain in the roots a diffusion pressure lower than in the surrounding soil.[3]

---

* Cons. Eng., Berkeley, Calif.

1. R. K. Schofield, The pF of the Water in Soil. Trans. Third International Congress of Soil Science, Oxford, England, 1935, Vol. 2, pp. 38-48.
2. "Soil Mechanics for Road Engineers," H. M. Stationery Office, London, 1952, pp. 300-304.
3. J. C. Fisher, Fracture of Liquids: Nucleation Theory Applied to Bubble Formation, Scientific Monthly, June, 1949, p. 415.

Professor Jumikis, who works on the basis of the thermo-osmosis concept, rightly explains that large scale experiments should be done to prove the practical significance of thermo-osmosis in highway engineering. The writer is familiar with the interesting work on thermo-osmosis by Prof. H. F. Winterkorn and pays high tribute to it. He believes, however, that there should be some limitations as to the use of the thermo-osmosis concept both in theory and in practice. This belief is partly based on the results of some recently published research work,[4] and on the attitude of J. R. Jennings, who, at the present time (1954) considers the thermo-osmosis phenomena in South Africa with reservation.[5]

Professor Jumikis' Reference 10 should be widened to include the most recent work by the Swiss professor R. Ruckli. In fact, Ruckli extended considerably his previous work and made the results available in the form of a book.[6]

The writer noticed with some adverse feeling that the temperature profile in Fig. 1 is termed "tautochrone." Generally the use of pseudo-Greek terms, which are mostly non-essential and often poorly coined, is hardly welcome in engineering.

Professor Jumikis should be commended for having attacked a difficult problem and obtained good results with apparently modest laboratory equipment.

4. L. A. Du Bose, A Full Scale Investigation of the Thermo-Osmosis Hypothesis, Proc., 3rd Conference on Soil Mechanics and Foundation Engineering, Zurich, Switzerland, 1953, Vol. 1, pp. 8-12.
5. J. E. Jenkins, The Heaving of Buildings on Dessicated Clay, ibid., Vol. 1, pp. 390-396. Also a discussion, ibid., Vol. 3, pp. 156-157.
6. Ruckli, R., "Der Frost im Baugrund," Springer-Verlag, Vienna, 1950.